$(\pi/2)$

	0	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	α

$Pa = N/m^2$ $\sin\pi = 0$ $\cos\pi \Rightarrow \text{Odd} = -1$

$\text{Even} = 1$

$Mpa = N/mm^2$ $\sin\pi/2 = 1$ $\cos\pi/2 = 0$

$1\pi/2$	$3\pi/2$	$5\pi/2$	$7\pi/2$
1	-1	1	-1

1KG = 9.81 N

1000KG = 1T

1 unit = 100 cft < Volume

1 inch = 2.54 cm

1 feet = 30.48 cm

1 cent = 435.6 Sqft

1 cent = $40.47 m^2$

1 ground = 2400 Sq.ft

1 square metre = 10.76 sq.ft

1 acer = 100 cent

1 hect = 2 1/2 acer.

Engineering Mechanics:

It is the Science that describes and predicts the effect of forceses on objects at rest or in motion

Classification

- Engineering Mechanics
  - Mechanics of Solids
    - Mechanics of rigid bodies
      - Statics
      - Dynamics
        - Kinematics
        - Kinetics
    - Mechanics of deformable bodies
      - Strength of Materials
  - Mechanics of fluids
    - * Ideal fluids
    - * Viscuous fluids
    - * Compressible fluids

Statics:-

It is the study of Effect of forces on rigid body which are at rest.

Dynamics:

It is the Study of Effect of forces on rigid body which are in motion

Kinematics:

It is the Study of a body in motion when the forces which cause the motion are not Considered.

Kinetics:

It is the Study of a body in motion when the forces which cause the motion are Considered.

Idealisation in Mechanics:-

Particle:-

It is assumed that the size of the body is negligible and it is irrelavant to the description of motion of the body. A particle is a body whose mass is Concentrated at a point.

It is a matter having Considerable mass and negligible dimension. / Space

Example: Sun, Earth Etc...

Rigid body :-

It is assumed that the distance between any two points of a body does not change under the action of applied force

It is a matter having Considerable mass and dimension.

Example : trusses , frames , beams Etc...

Matter :-

Matter is a Substance which occupies Space It is made up of atoms and molecules.

Mass :

The quantity of matter possessed by a body is Called mass. Mass of a body does not Vary with the location and orientation of the body

Force :- $F = ma$

It is defined as the ability to translate a body into action or as the action of one body on another

The Characteristics of force are ,

* Magnitude
* Direction
* point of application (line of action)

It is a Vector quantity

Weight :-

The weight of a body is the force with which the Earth attract the body towards its centre.

By Newton's Second law of motion, the weight of a body is given by

$$W = mg$$

where m = mass of the body
g = acceleration due to gravity ie $9.81\ m/s$

Mass	Weight
It is the Quantity of matter contained in a body	It is the force with which the body is attracted towards the Centre of Earth
It is Constant at all places	It is not constant at all places
It resists motion in body	It provides moition in body
It is a Scalar quantity	It is a Vector quantity
It is never Zero	It is Zero at the Centre of
It is measured in Kg, both in M.K.S and S.I units	It is measured in Kgwt in M.K.S units and Newton (N) in S.I units

Space:

It is the region that Extents in all direction and Contains Everything in it.

It is used to fix the position of a point in relation to a reference point known as origin

(or)

It is defined as the geometric region occupied by a body whose position

Time

It is used to measure the Sequence of Events. It is used in dynamics but not in Statics.

Inertia:

The resistance offered by a matter to any Change of its state of motion is Called inertia.

Scalar Quantity:

A physical quantity of matter which requires only magnitude for its Complete description is known as Scalar quantity

Example: distance, area, Volume, mass, work, Power, Energy, time, density, Speed etc.

Vector Quantity:

A physical Quantity of matter which requires both magnitude and direction for its Complete description is Called as Vector Quanity.

Example: Force, displacement, velocity, acceleration, moment, Couple, torque, impluse, momentum, weight etc.

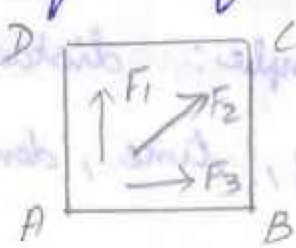
Classification of Force System:

Force

- Coplanar (plane)
  - Collinear
  - Concurrent
  - parallel
  - non-Concurrent, Non-parallel
- Non Coplanar (Space)
  - Concurrent
  - parallel
  - non-Concurrent, non-parallel

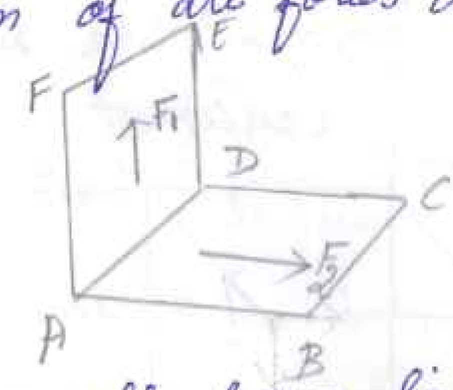
Co-planar Force:

Line of action of all forces lie on the Same plane



Non-Coplanar Force:

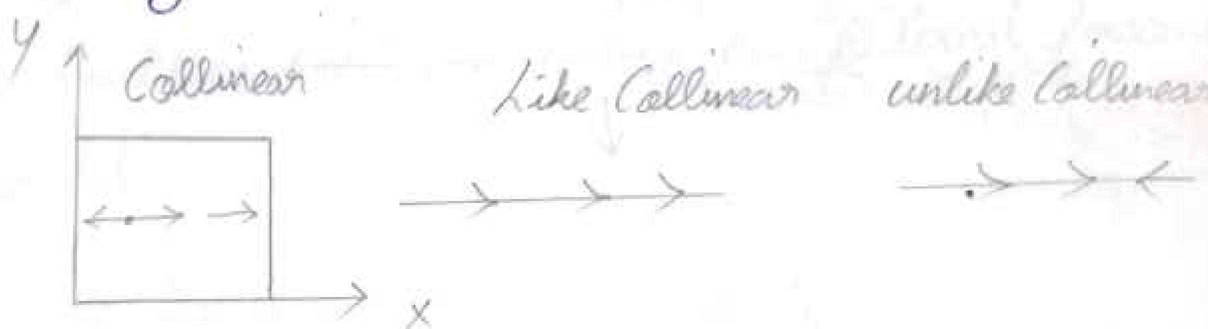
Line of action of all forces donot lie on the Same plane.



Collinear Force:

Line of action of all forces lie on the Same line is Called Collinear Force.

If they act in Same direction, they are Called like Collinear and if they act in opposite direction they are Called unlike Collinear.

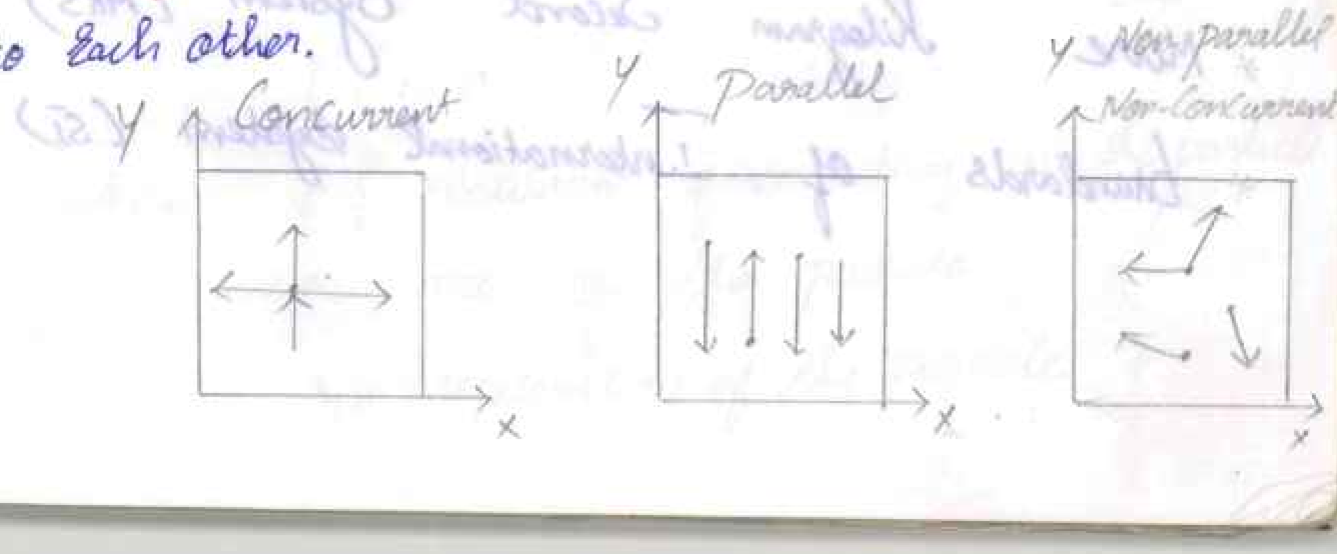


Concurrent force:-

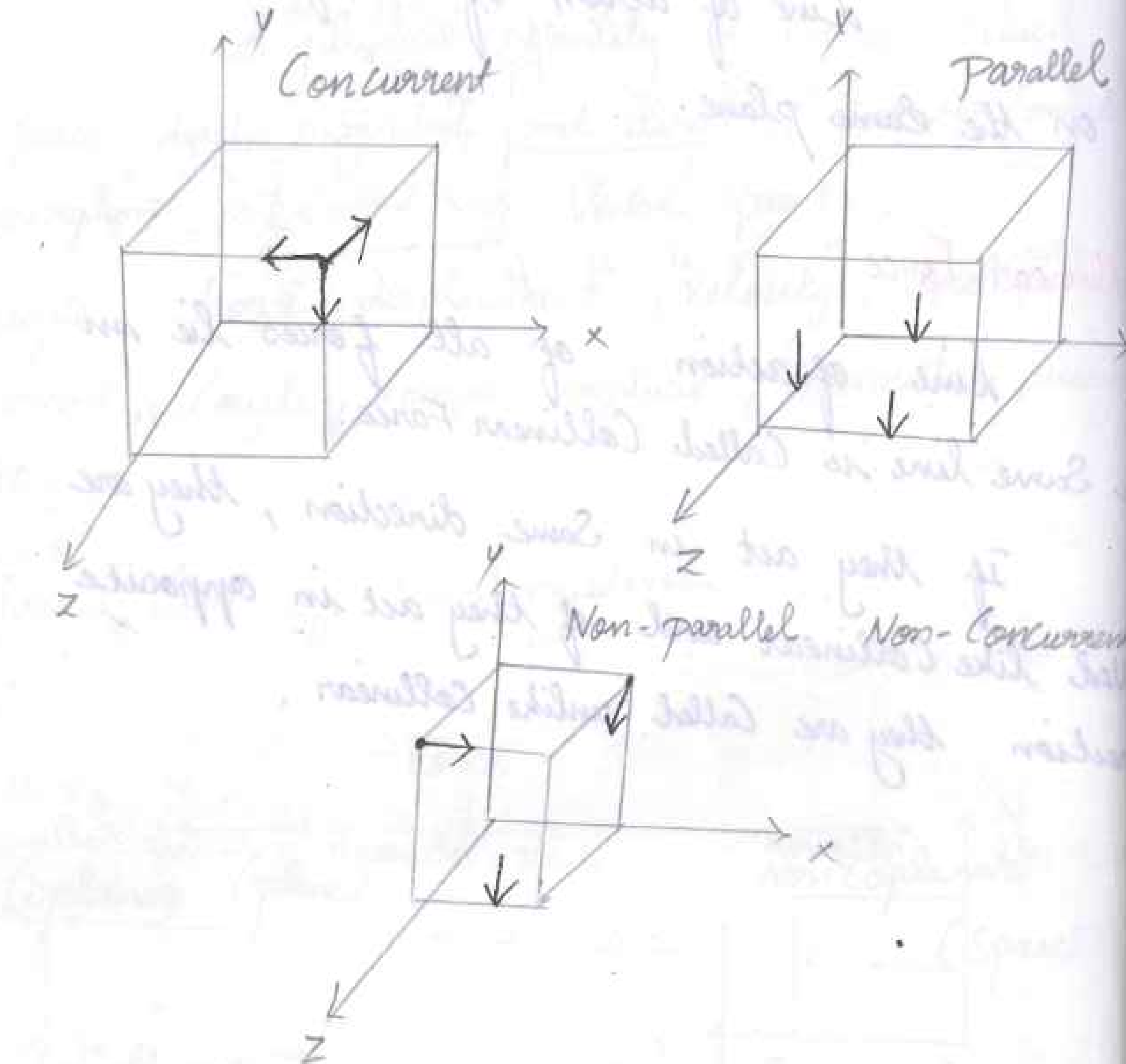
line of action of all forces passes throught Same point

Like parallel
Unlike parallel

Parallel forces: line of action of all forces parallel to Each other.



Non - Coplanar Force System.



System of units :-

* Foot pound Second System (FPS)
* Centimetre Gram Second System (CGS)
* Metre Kilogram Second System (MKS)
* Standards of International System (SI)

Fundametal Principle:

The basic laws of Mechanics are:-

* Newton's law of motion
* Newton's law of gravitation
* Parallelogram law
* Principle of Transmissibility

i) Newton's First law:

If a body has the state of rest or uniform motion, then it will Continue to have the Same state of Condition until and unless an External force influences it.

ii) Newton's Second law:

If a resultant force acting on a particle is not zero, the acceleration of the particle will be proportional to the magnitude and the direction of the resultant force

$$F = ma$$

where F = resultant force acting on the particle

m = mass of the particle

a = acceleration of the particle

iii) Newton's third law:-

To Every action, there is an Equal and opposite reaction. This means that the forces of action and reaction between the bodies in Contact have the Same magnitude and line of action but opposite in direction.

2) Newton's Law of Gravitation:

It States that the force of attraction between any two bodies is directly proportional to th masses and inversely proportional to the Square of the distance between them.

$$F = \frac{G m_1 m_2}{r^2}$$

where m_1 = mass of first body

m_2 = mass of Second body

r = distance between the Centre of bod

F = Force of attraction between the bodie

G = universal Constant known as the Constant of gravitation.

$$F = \frac{G m_1 m_2}{r^2}$$

Acceleration = $\frac{\text{Change in Velocity}}{\text{time taken}}$

$$G = \frac{F r^2}{m_1 m_2} = \frac{m \times a \times r^2}{m_1 m_2}$$

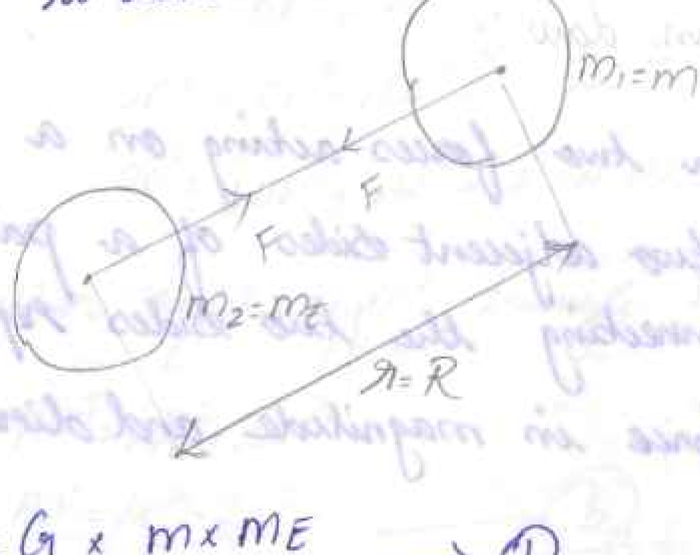
$$= \frac{\cancel{kg} \times \frac{m}{s^2} \times m^2}{\cancel{kg} \times kg}$$

$= m/s^2$

$$= \frac{m^3}{kg \cdot s^2}$$

$$G = (66.7 \pm 0.03) \times 10^{-12}\ m^3/kg \cdot s^2$$

weight is defined as the forces with which the body is attracted towards the earth's centre.



$$W = G \times \frac{m \times m_E}{R^2} \longrightarrow (1)$$

where, m = mass of the body

m_E = mass of the Earth = 5.9761×10^{24} kg

R = distance between the Centres of Earth and the body

(ie) Radius of the Earth $R = 6.37 \times 10^6$ m we know,

weight = mass × acceleration due to gravity

$$w = m \times g \rightarrow (2)$$

Eq (1) & (2)

$$\frac{G \times \cancel{m} \times m_E}{R^2} = \cancel{m} \times g$$

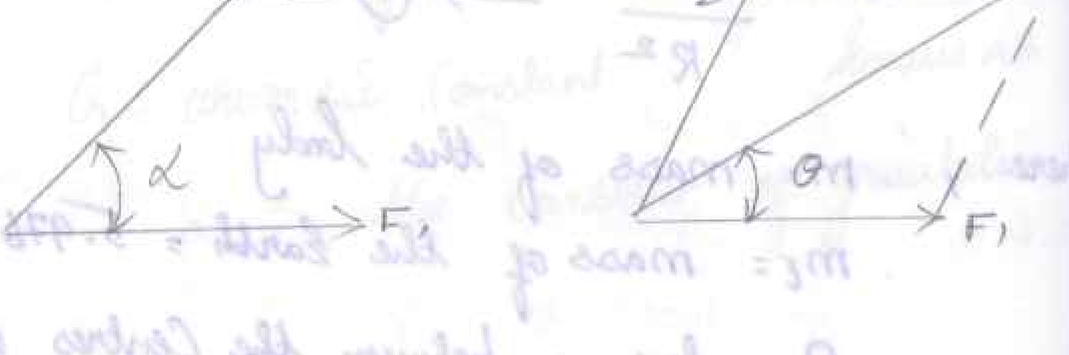
$$g = \frac{G \times m_E}{R^2} = \frac{66.7 \times 10^{-12} \times 5.9761 \times 10^{24}}{(6.37 \times 10^6)^2}$$

$$g = 9.8234\ m/s^2$$

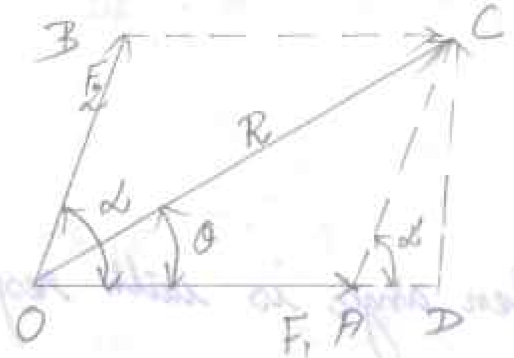
Conventionally we take, $\boxed{g = 9.81\ m/s^2}$

3) Parallelogram law :-

when two forces acting on a particle represented by two adjecent sides of a parallelogm the diagonal connecting the two sides represents the resultant force in magnitude and direction.



The relationship between F_1, F_2 and R are derived as follows.



$OA = \vec{F_1}$

$OB = \vec{F_2}$

$OC = \vec{R}$

The angle between two forces F_1 and F_2 be 'α'.

The resultant angle be 'θ'

From the above figure, we have right angled triangle ODC,

By Pythogorous theorem,

$$OC^2 = OD^2 + DC^2$$

$$OC^2 = (OA^2 + AD^2) + DC^2$$

$$OC^2 = (OA^2 + AD^2 + 2.OA.AD) + DC^2 \longrightarrow (a)$$

From the other right angled triangle ADC,

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \longrightarrow (b)$$

Sub 'b' Eq in 'a' eq

$$OC^2 = OA^2 + AC^2 - \cancel{DC^2} + 2.OA.AD + \cancel{DC^2}$$

$$OC^2 = OA^2 + AC^2 + 2.OA.AD$$

$OC = R$

$OA = F_1$

$AD = F_2 \cos\alpha$

$AC = F_2$

$\cos\alpha = \frac{AD}{AC}$

$AD = AC \cos\alpha$

Now, $= F_2 \cos\alpha$

$OC^2 = OA^2 + AC^2 + 2.OA.AD$

$R^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos\alpha$

Resultant;

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\alpha}$$

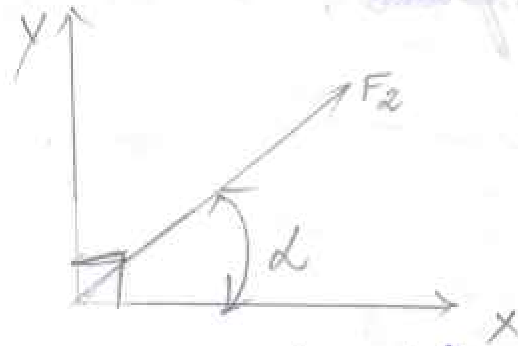
Resultant angle :

$\tan\theta = \frac{DC}{OD}$

$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$

$\theta = \tan^{-1}\left(\frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}\right)$

Resolving Forces



when angle is with respect to x-axis

$X = \cos$

$Y = \sin$

X - Component of F_2

$F_2 \cos\alpha$

Y - component of F_2

$F_2 \sin\alpha$

$OD = OA + AD$

$\boxed{OD = F_1 + F_2 \cos\alpha}$

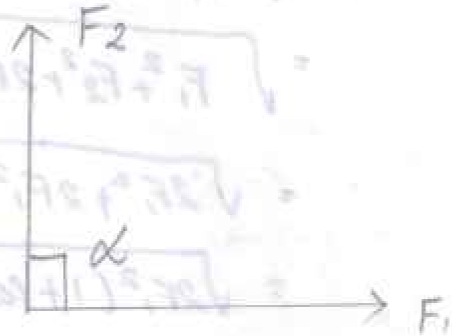
$\sin\alpha = \frac{DC}{AC}$

$DC = AC \sin\alpha$

$\boxed{DC = F_2 \sin\alpha}$

Special Cases:-

Case 1: If F_1 and F_2 are at right angles



$\alpha = 90^\circ$

$\sin\alpha = \sin 90^\circ = 1$

$\cos\alpha = \cos 90^\circ = 0$

$R = \sqrt{F_1^2 + F_2^2}$

$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) \Rightarrow \tan\theta = \frac{F_2}{F_1}$

Case 2: If F_1 and F_2 are Collinear and acting in the same direction

$\alpha = 0^\circ$

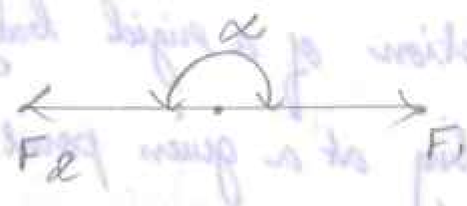
$\sin\alpha = \sin 0 = 0$

$\cos\alpha = \cos 0 = 1$

$R = \sqrt{F_1^2 + F_2^2 + 2.F_1F_2} = \sqrt{(F_1+F_2)^2} = (F_1+F_2)$

$\tan\theta = 0$

Case 3: If F_1 and F_2 are Collinear and acting in the opposite direction



$\alpha = 180^\circ$

$\sin\alpha = \sin 180^\circ = 0$

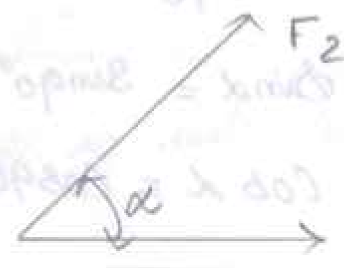
$\cos\alpha = \cos 180^\circ = -1$

$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2} = \sqrt{(F_1-F_2)^2} = (F_1-F_2)$

$\tan\theta = 0.$

Case 4: If F_1 and F_2 are at an angle 'α' between and $F_1 = F_2$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$



$$= \sqrt{F_1^2 + F_2^2 + 2F_1^2\cos\alpha}$$

$$= \sqrt{2F_1^2 + 2F_1^2\cos\alpha}$$

$$= \sqrt{2F_1^2(1+\cos\alpha)}$$

$$= \sqrt{2F_1^2\left(2\cos^2\frac{\alpha}{2}\right)}$$

$$R = 2F_1\cos\frac{\alpha}{2}$$

$$\tan\theta = \frac{F_1\sin\alpha}{F_1(1+\cos\alpha)} = \frac{\sin\alpha}{2\cos^2\frac{\alpha}{2}} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}$$

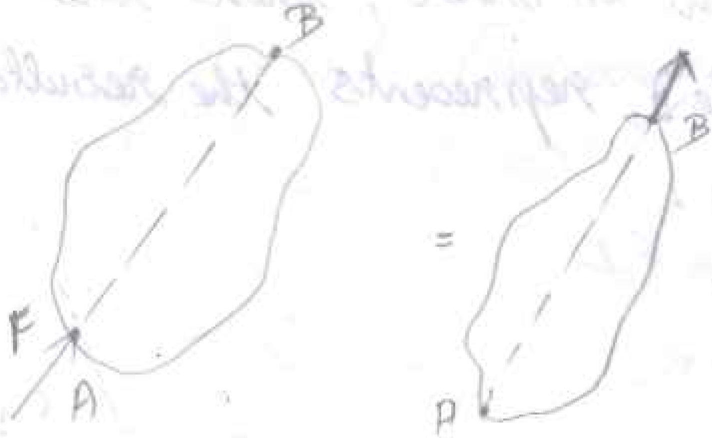
$$= \frac{\sin\alpha/2}{\cos\alpha/2}$$

$$\tan\theta = \tan\frac{\alpha}{2}$$

$$\theta = \frac{\alpha}{2}.$$

4) Principle of transmissibility :-

The Condition of an Equilibrium or motion of a rigid body remains unchanged, if a force acting at a given point is replaced by a force of same magnitude and direction but different point, provided that they have same line of action.

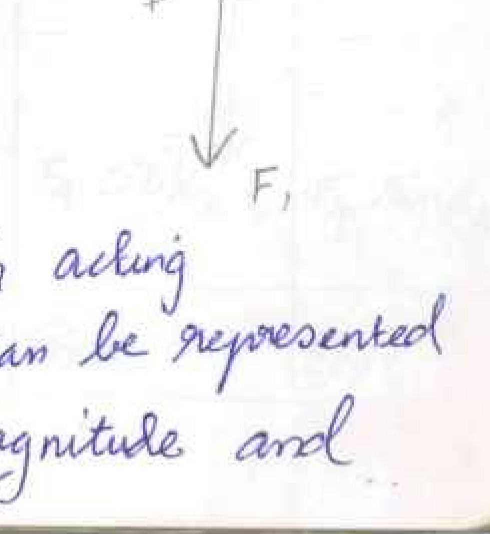


Law of Mechanics

* Lami's theorem
* Triangle law of forces
* Polygon Law.

Lami's theorem:-

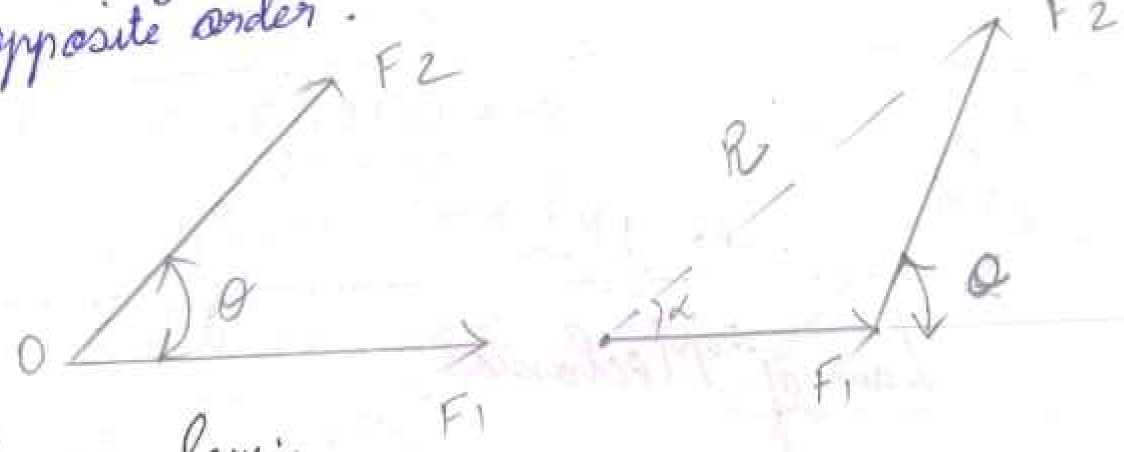
It States that, if three Co-planar forces acting at a point be in Equilibrium then each force is proportional to the Sine of the angle between the other two.



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}.$$

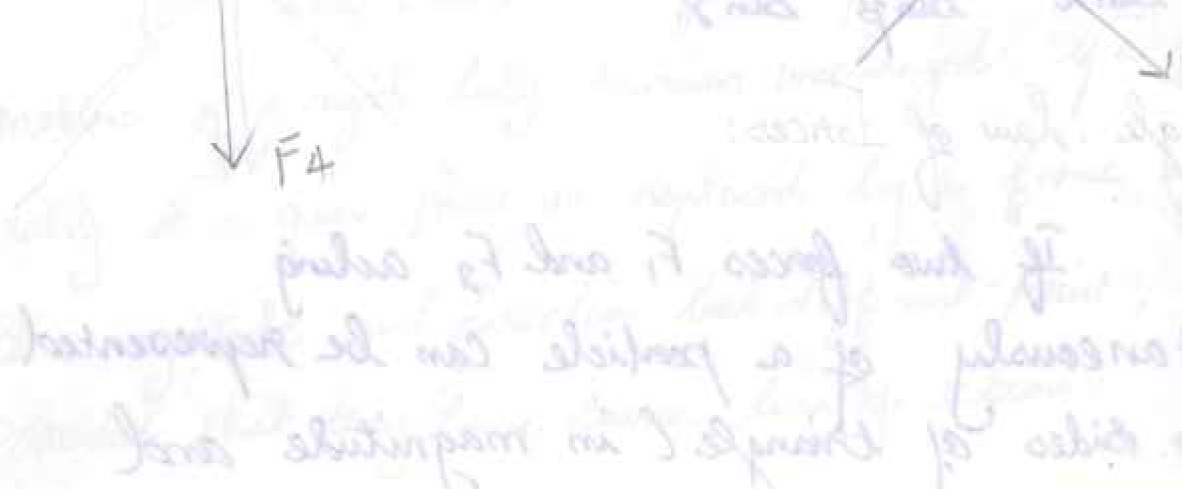
Triangle law of Forces:

If two forces F_1 and F_2 acting Simentaneously of a particle can be represented by two sides of triangle (in magnitude and direction) taken in order, then the third Side (closing side) represents the resultant in opposite order.

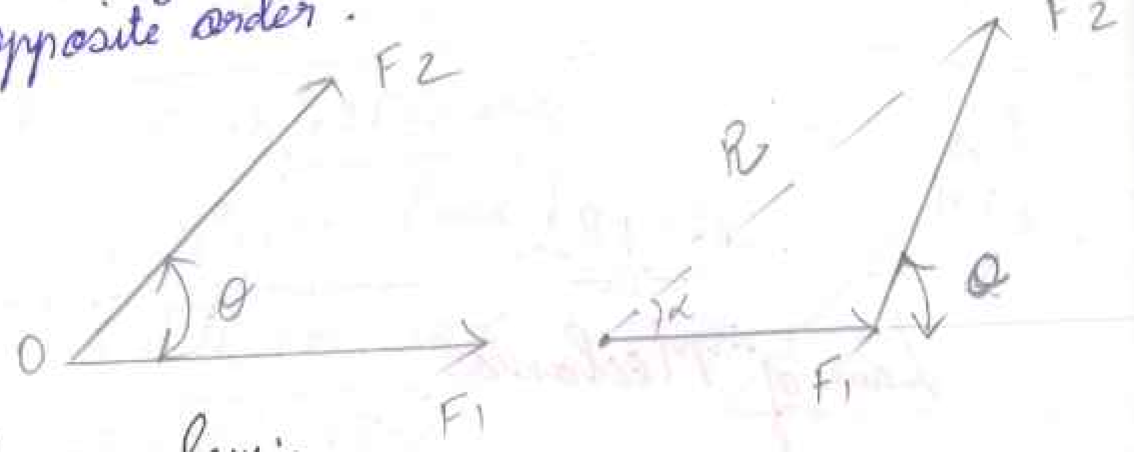


Polygon law:

If a number of Concurrent force acting Simentaneously on a particle are represe in terms of magnitude and direction by the sides of polygon taken in order, then the resultant of System forces is represented by Closing sides of the polygon taken in the reverse order



direction) taken in order, then the third Side (closing side) represents the resultant in opposite order.



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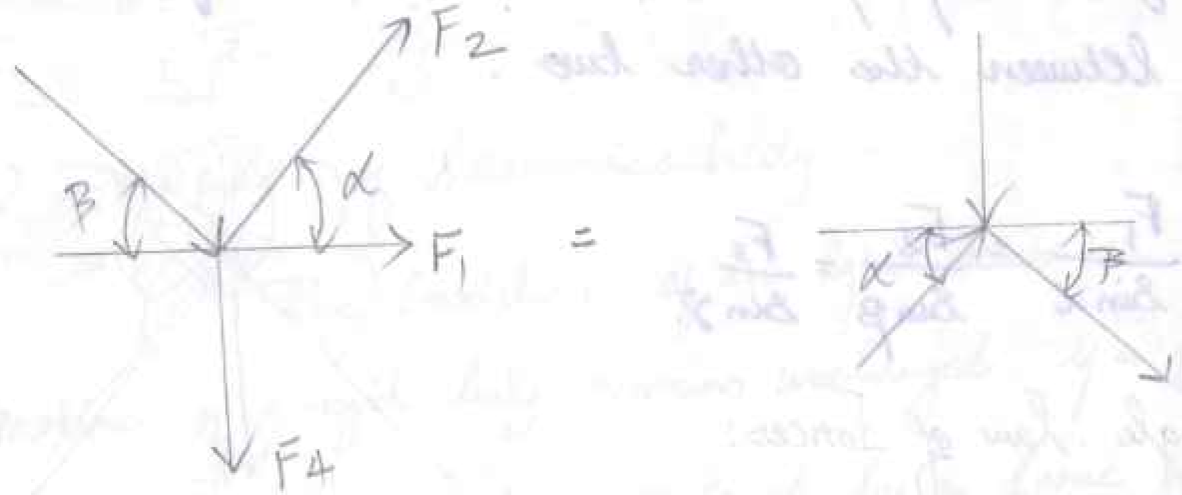
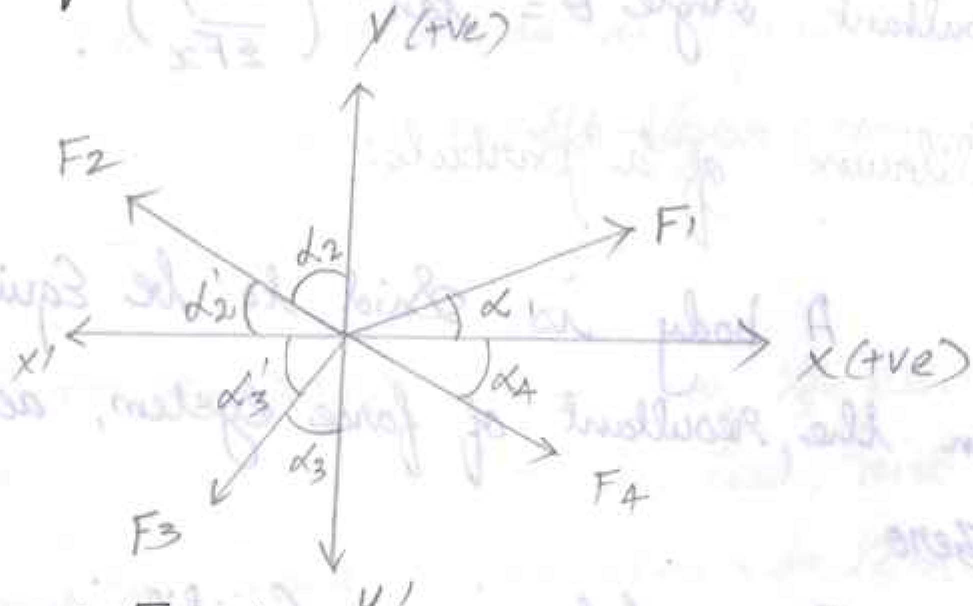
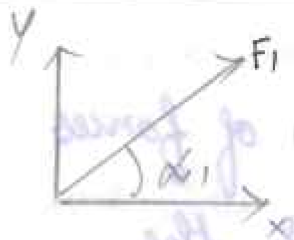
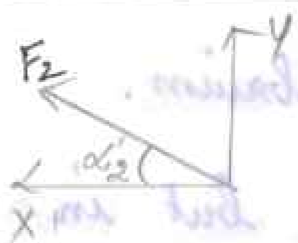
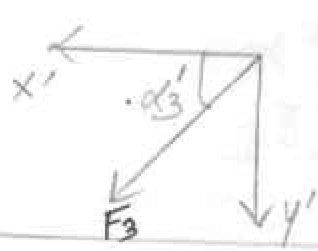
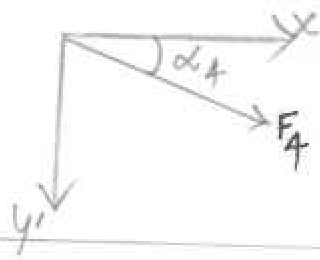


Resultant of Several Concurrent Forces:



Components of Force:-

Force	Magnitude F	Direction α	X-component F_x	Y-component F_y
	F_1	α_1	$F_1 \cos\alpha_1$	$F_1 \sin\alpha$
	F_2	α_2'	$-F_2 \cos\alpha_2'$	$F_2 \sin\alpha_2'$
	F_3	α_3'	$-F_3 \cos\alpha_3'$	$-F_3 \sin\alpha_3'$
	F_4	α_4	$F_4 \cos\alpha_4$	$-F_4 \sin\alpha_4$
			$\Sigma F_x =$	$\Sigma F_y =$

Resultant $(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

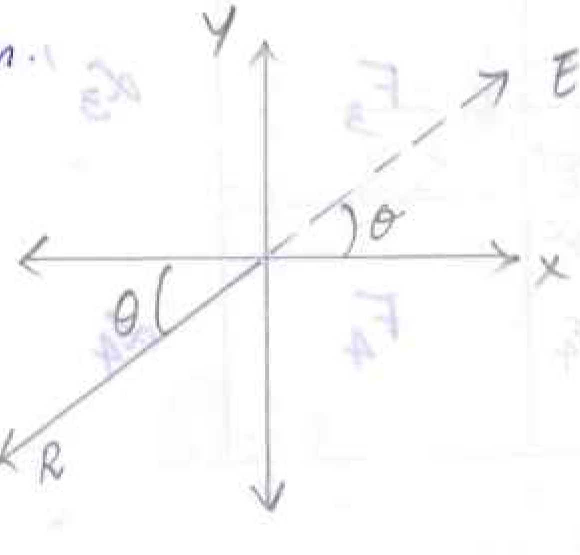
Resultant Angle $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$.

Equilibrium of a particle:

A body is Said to be Equilibrium, when the resultant of force System, acting on it is Zero

If a body is in Equilibrium, it will Continue to remain in its State of rest or uniform motion.

Equilibrant:- Equilibrant of a System of forces is a Single force which acts along with the other forces to keep the body in Equilibrium. It is Equal in magnitude of resultant but in opposite in direction.



Resultant $(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

Resultant Angle $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$.

Equilibrium of a particle:

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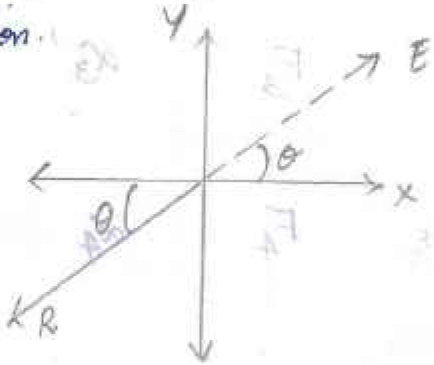


Equations of Equilibrium:

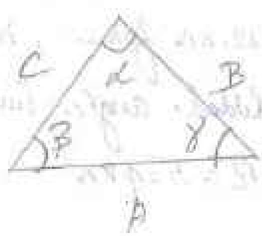
when a particle is in Equilibrium the resultant force is Zero or the Vector (resultant) is Zero $\vec{R} = 0$

Since the resultant is Zero, the Sum of forces acting along x-direction is Zero, and Sum of forces acting along y-direction is Zero

$\Sigma F_x = 0$

$\Sigma F_y = 0$

Triangle law of forces:



Sine law is written as

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Cosine law is written as

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$
$$B^2 = A^2 + C^2 - 2AC\cos\beta$$
$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

* Two Concurrent forces acts at an angle of 30°. The resultant forces is 15N and one of the forces is 10N. Find the other force.

Sol:

$R = 15N$; $P = 10N$; $\theta = 30°$; $Q = ?$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$15^2 = 10^2 + Q^2 + (2 \times 10 \times Q\cos 30°)$

$125 - Q^2 - 17.32Q = 0$ Bring Everything right side

$Q^2 + 17.32Q - 125 = 0$

By using the calc $x_1 = 5.48N$ $x_2 = -22.8N$

Reject $-22.8N$ force, because if the force is reversed the included angle will be 150° not 30°.

Hence $Q = 5.48N$

* Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}N$, But if they act at 60°, their resultant is $\sqrt{13}N$.

Sol:

Case 1) $\theta = 90°$

Case 2) $\theta = 60°$

$R^2 = P^2 + Q^2 + 2PQ\cos 90°$

$R^2 = P^2 + Q^2$

$10 = P^2 + Q^2 \longrightarrow (1)$

$R^2 = P^2 + Q^2 + 2PQ\cos 60°$

$R^2 = P^2 + Q^2 + PQ$

$13 = P^2 + Q^2 + PQ \longrightarrow (2)$

Sub the (1) Eq in (2)

$13 = P^2 + Q^2 + PQ$

$13 = 10 + PQ$

$\therefore PQ = 3$

by using $(P+Q)^2 = P^2 + Q^2 + 2PQ$

$= 10 + (2 \times 3)$

$= 16$

$P + Q = \sqrt{16}$

$P + Q = 4 \longrightarrow (3)$

by using $(P-Q)^2 = P^2 + Q^2 - 2PQ$

$= 10 - (2 \times 3)$

$= 4$

$P - Q = \sqrt{4}$

$P - Q = 2 \longrightarrow (4)$

by solving (3) & (4) Eq we get

$P = 3N$ and $Q = 1N$.

forces of magnitude 10N and 8N are acting @ a point. If the angle between the two forces is 60°. Determine the magnitude of the resultant force.

Sol:

Given, Force $P = 10N$; Force $Q = 8N$

Angle between the two forces, $\theta = 60°$

Magnitude of Resultant Force, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

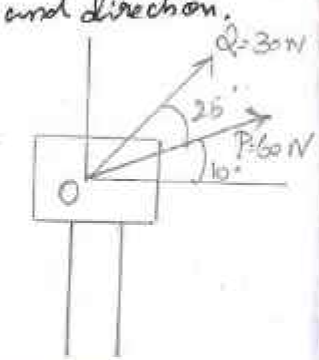
$$R = \sqrt{10^2 + 8^2 + 2\times10\times8\cos60°}$$

$$= \sqrt{100 + 64 + 80}$$

$$= 15.62\,N.$$

$2\times10\times8\cos60°$
$= 160\cos60°$
$= 160\times\frac{1}{2}$
$= 80$

Two forces P and Q act on a bolt as shown in fig. Determine the resultant in magnitude and direction.



Sol: Given $P = 60N$; $Q = 30N$

Angle between the two forces, $\theta = 25°$

Magnitude of the Resultant, R

$$= \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{60^2 + 30^2 + 2\times60\times30\cos25°}$$

$$= \sqrt{3600 + 900 + (3600\times0.906)}$$

$$= 88.10\,N$$

Direction of the resultant, $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta} = \dfrac{30\sin25}{60+30\cos25}$

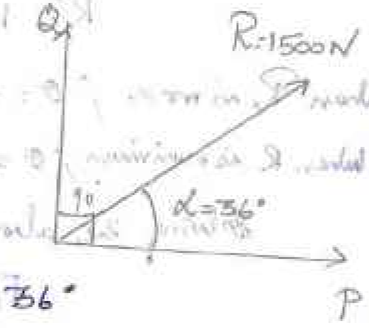
$$\frac{\quad}{87.19} = 0.145$$

$$\alpha = \tan^{-1}(0.145) = 8.25°$$

The resultant of two concurrent forces is 1500N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the force. Find the magnitude of each force.

Sol:

Given



Resultant, $R = 1500\,N$

Angle b/w the forces, $\theta = 90°$

Angle made by resultant with one force, $\alpha = 36°$

Let P and Q are two forces

Using Eq, $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$\sin90° = 1$
$\cos90° = 0$

$$\tan36° = \frac{Q\sin90}{P + Q\cos90} = \frac{Q}{P}$$

$$Q = P\tan36°$$

$$Q = 0.726P \longrightarrow (1)$$

Using equation, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$1500^2 = P^2 + (0.726P)^2 + 2P(0.726P)\times\cos90°$$

(0)

$$1500^2 = P^2 + (0.726P)^2$$

$P^2 + 0.527P^2$

$$1500^2 = 1.527P^2$$

$$P = \sqrt{\frac{1500^2}{\quad}} = 1213.87\,N$$

$P = 1213.87N$

$Q = 0.726\times1213.87$
$= 881.27\,N.$

The greatest and least resultant of two forces acting on a particle are 35 KN and 5 KN respectively. If 25 KN is the magnitude of the resultant for the given system of forces F_1 and F_2, prove that the forces are at right angles.

Resultant, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

when R is max, ($\theta = 0°$) i.e. $35 = F_1 + F_2$

when R is minimum ($\theta = 180$) i.e. $5 = F_1 - F_2$

Solving the above two equation

$$F_1 = 20\text{ kN} \quad \& \quad F_2 = 15\text{ kN}$$

If the resultant of forces F_1, F_2 acting @ an angle 'θ' is 25 kN

$$\therefore 25^2 = 20^2 + 15^2 + 2(20)(15)\cos\theta$$

$$625 = 400 + 225 + 600\cos\theta$$

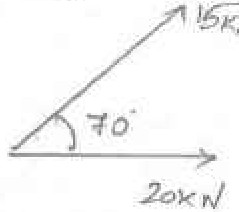
$$\cos\theta = 0$$

$$\theta = 90°$$

Thus F_1 & F_2 are at right angles to each other when the resultant is 25 kN.

$$A^2 = B^2 + C^2 - 2AB\cos\theta \leftarrow \text{Triangle law of Cos}$$

If two forces $F_1 = 20$ kN and $F_2 = 15$ kN act on a particles as shown in fig. Find their resultant by (a) parallelogram law and (b) Triangle law.



Sol:

a) Parallelogram law

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{\underbrace{20^2 + 15^2 + 2(15)(20)\cos 70}_{205.212}}$$

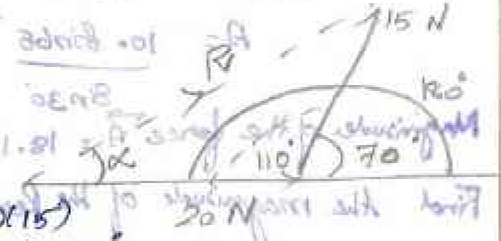
$$R = 28.81\text{ kN}$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} = \frac{15\sin 70}{20 + 15\cos 70} = \frac{14.09}{25.13}$$

$$\tan\alpha = 0.56$$

$$\alpha = 29.28°$$

b) Triangle law



$$R^2 = 20^2 + 15^2 - 2(20)(15)\cos 110°$$

$$R^2 = 400 + 225 - 600 \times (-0.34)$$

$$= 400 + 225 + 204$$

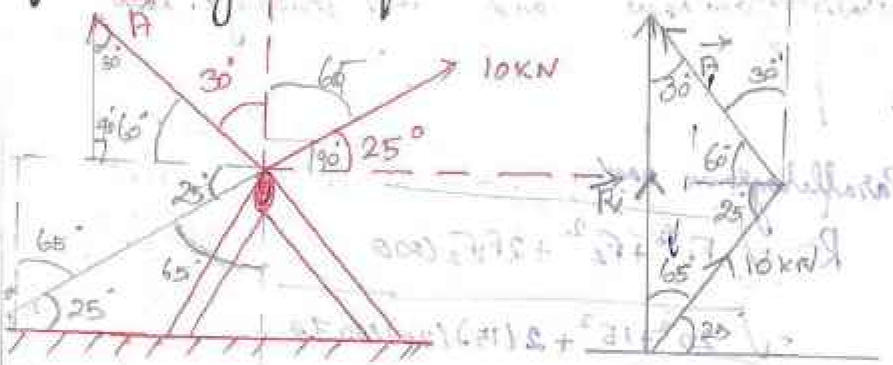
$$R = 28.79\text{ kN.}$$

Applying Sine law,

$$\frac{R}{\sin 110} = \frac{15}{\sin\alpha} \qquad R\sin\alpha = 15\sin 110 = 14.09$$

$$\alpha = 29.31°$$

Find the magnitude of $\vec{A}$ in the system shown in fig. in order that the resultant is vertical. Also find the magnitude of the resultant.



Sol.

Find the magnitude of $\vec{A}$

Applying the law of sines,

$$\frac{A}{\sin 65^\circ} = \frac{10}{\sin 30^\circ}$$

$$A = \frac{10 \cdot \sin 65^\circ}{\sin 30^\circ} = \frac{10 \times 0.906}{0.5} = 18.12\,kN$$

Magnitude of the force $\vec{A}$ = 18.13 kN.

Find the magnitude of the Resultant.

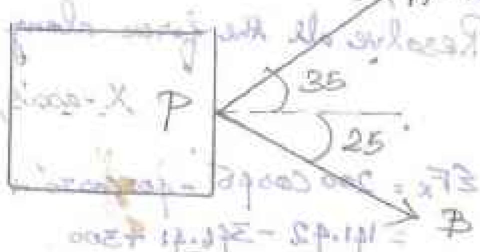
Applying the law of sine

$$\frac{R}{\sin 85^\circ} = \frac{10}{\sin 30^\circ}$$

$$R = \frac{\sin 85^\circ \times 10}{\sin 30^\circ} = 19.92\,kN$$

Magnitude of the resultant = 19.92 kN

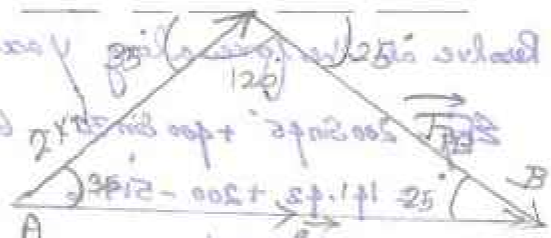
A cart is pulled by two ropes as shown in fig. The tension in rope PA is 2 kN. The resultant acts along the axis of the cart. Determine the tension in rope PB and the magnitude of the resultant of two forces.



Sol:

let $\vec{T}_{PB}$ be the tension in rope PB.

Applying the law of sines.



$$\frac{T_{PB}}{\sin 35^\circ} = \frac{2}{\sin 25^\circ}$$

$$T_{PB} = \frac{\sin 35^\circ \times 2}{\sin 25^\circ}$$

$T_{PB} = 2.71\,kN$.

Find the magnitude of the resultant

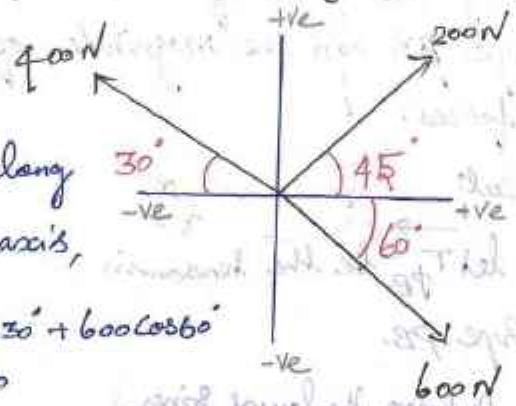
Applying the law of sines.

$$\frac{R}{\sin 120^\circ} = \frac{2}{\sin 25^\circ}$$

$$R = \frac{2 \times \sin 120}{\sin 25^\circ} = \frac{1.73}{0.423}$$

$R = 4.09\,kN$

Three coplanar concurrent forces are acting as shown in fig. Determine the resultant in magnitude and direction.



Sol:

Resolve all the forces along X-axis,

$$\Sigma F_x = 200\cos 45° - 400\cos 30° + 600\cos 60°$$
$$= 141.42 - 346.41 + 300$$
$$= 95.01\,N.$$

Resolve all the force along y axis,

$$\Sigma F_y = 200\sin 45° + 400\sin 30° - 600\sin 60$$
$$= 141.42 + 200 - 519.6$$
$$= -178.18\,N$$

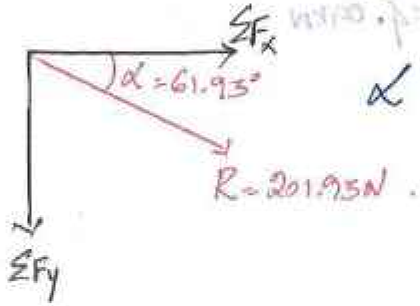
Magnitude of resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(95.01)^2 + (-178.18)^2}$$
$$= 201.93\,N.$$

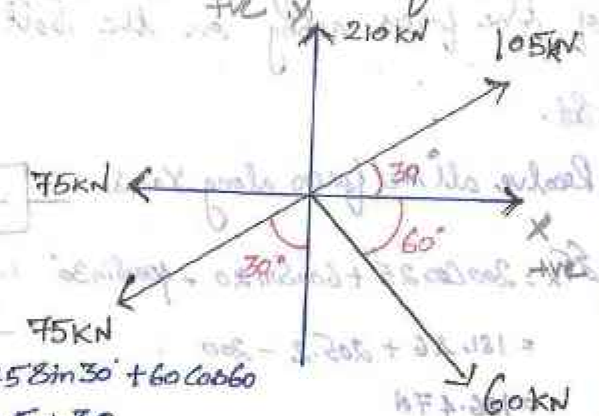
Direction of Resultant, $\tan\alpha = \Sigma F_y / \Sigma F_x$.

$$\alpha = \tan^{-1}\left(\frac{178.18}{95.01}\right)$$

$$\alpha = 61.93°.$$



If five forces act on a particle as shown in fig. Determine the magnitude and direction of the resultant.



Sol:

Resolve all the forces along X-axis

$$\Sigma F_x = 105\cos 30° - 75 - 75\sin 30° + 60\cos 60$$
$$= 90.93 - 75 - 37.5 + 30$$
$$= 8.43\,KN$$

Resolve all the forces along Y axis

$$\Sigma F_y = 105\sin 30 + 210 - 75\cos 30 - 60\sin 60$$
$$= 52.5 + 210 - 64.95 - 51.96$$
$$= 145.59\,KN$$

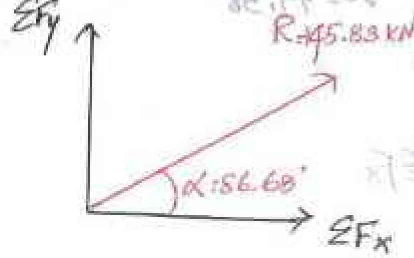
Magnitude of resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(8.43)^2 + (145.59)^2}$$
$$= 145.83\,KN.$$

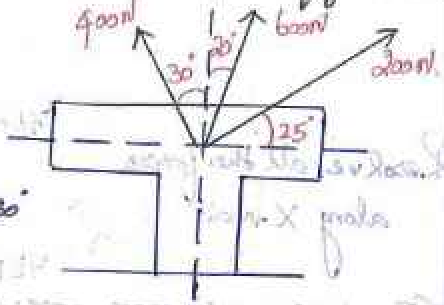
Direction of resultant, $\tan\alpha = \dfrac{\Sigma F_y}{\Sigma F_x}$

$$\alpha = \tan^{-1}\left(\frac{145.59}{8.43}\right)$$

$$\alpha = 86.68°$$



Determine the magnitude and direction of the resultant of the forces acting on the bolt as shown in fig.



Sol.

Resolve all the forces along X axis,

$\Sigma F_x = 200\cos 25° + 600\sin 20° - 400\sin 30°$

$= 181.26 + 205.2 - 200$

$= 186.47N$

Resolve all the forces along Y axis,

$\Sigma F_y = 200\sin 25 + 600\cos 20° + 400\cos 30°$

$= 994.74 N.$

Magnitude of Resultant,

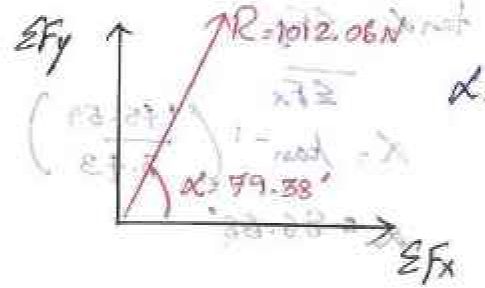
$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(186.47)^2 + (994.74)^2}$

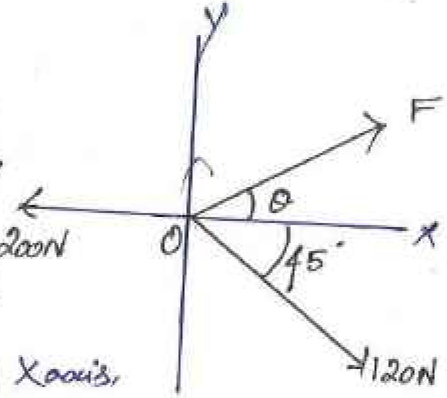
$= 1012.06N$

Direction of Resultant, $\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{994.74}{186.47}$

$\alpha = 79.38°$



The Force system shown in fig has a resultant of 100N pointing up along the y axis. Compute the values of F and θ required to given this resultant.



Sol:-

Given that the resultant of 100N pointing up along y axis, therefore

$\Sigma F_y = R$ and $\Sigma F_x = 0$

Resolve all the forces along X axis,

$\Sigma F_x = F\cos\theta + 120\cos 45° - 200$

$0 = F\cos\theta + 84.85 - 200$

$F\cos\theta = 200 - 84.85$

$F\cos\theta = 115.15 \longrightarrow (i)$

Resolve all the forces along Y axis

$\Sigma F_y = F\sin\theta - 120\sin 45°$

$100 = F\sin\theta - 84.85$

$F\sin\theta = 184.85 \longrightarrow (ii)$

Divide Eq (ii) by (i)

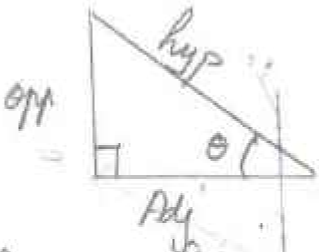
$\frac{F\sin\theta}{F\cos\theta} = \frac{184.85}{115.15} = 1.60$

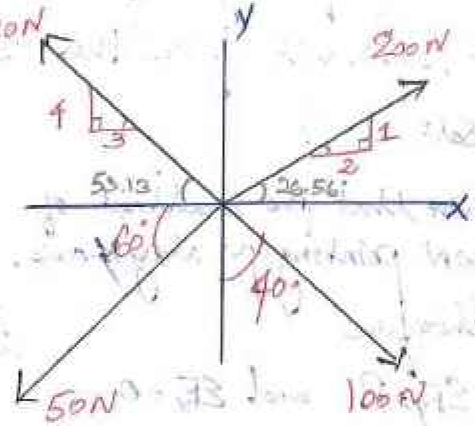
$\tan\theta = 1.60 \qquad \theta = 58.0$

Sub 'θ' in Eq (i)

$F = \frac{115.15}{\cos 58.0} = 217.30N.$

Determine the resultant both in magnitude and direction of the four forces acting on the body as shown in fig.





$\sin\theta = opp / hyp$

$\cos\theta = Adj / hyp$

$\tan\theta = opp / Ady = \sin/\cos$

$\tan\theta = \frac{4}{3}$; $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

$\tan\theta = \frac{1}{2}$; $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$

Resolve all the force along x axis,

$\Sigma F_x = 200\cos 26.56° - 120\cos 53.13 - 50\cos 60 + 100\sin 40° = 146.17N$

$\Sigma F_y = 200\sin 26.56° + 120\sin 53.13° - 50\sin 60° - 100\cos 40°$

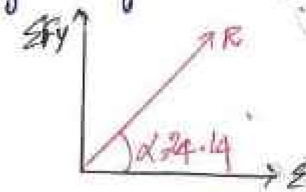
$\Sigma F_y = 89.42 + 95.99 - 43.30 - 76.60$

$= 65.51N$

Magnitude of Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(146.17)^2 + (65.52)^2}$

$= 160.18N.$



Direction of resultant, $\tan\alpha = \Sigma F_y / \Sigma F_x = \left(\frac{65.51}{146.17}\right)$

$\alpha = 24.14°$

Determine the magnitude and direction of the Resultant of the following Set of forces acting on a body.

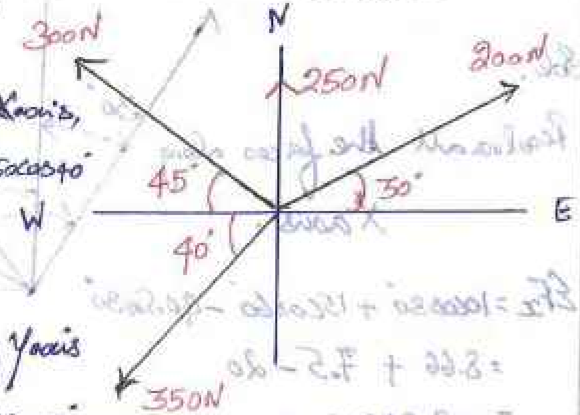
(i) 200N inclined 30° with East towards North

(ii) 250N towards the North

(iii) 300N towards North west @45° inclination

(iv) 350 N inclined @40° with west towards South

Sol



Resolve all the force along X axis,

$\Sigma F_x = 200\cos 30° - 300\cos 45° - 350\cos 40°$

$= 173 - 212.13 - 268.11$

$= -307.04N$

Resolve all the forces along Y axis

$\Sigma F_y = 200\sin 30° + 250 + 300\sin 45° - 350\sin 40°$

$= 100 + 250 + 212.13 - 224.97$

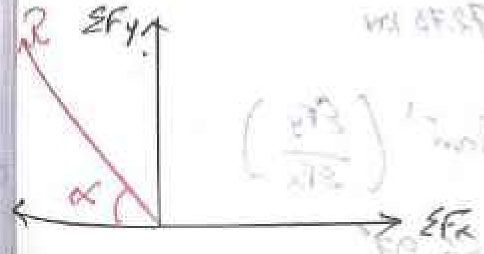
$= 337.16N$

Magnitude of Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-307.04)^2 + (337.16)^2}$

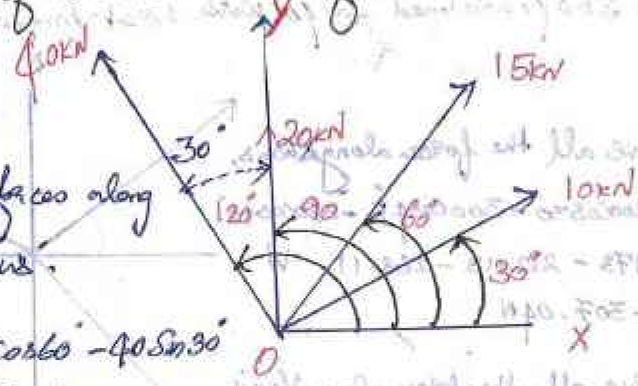
$= 456.0 N$

Direction of Resultant, $\tan\alpha = \Sigma F_y / \Sigma F_x = \frac{337.16}{307.04}$



$\alpha = 47.68°$

Four forces of magnitude 10KN, 15KN, 20KN and 40KN are acting @ a point O as shown in fig. The angle made by 10KN, 15KN, 20KN and 40KN with X axis are 30°, 60°, 90° and 120° respectively. Find the magnitude and direction of the resultant force.



Sol:

Resolve all the forces along X axis.

$\Sigma F_x = 10\cos 30° + 15\cos 60° - 40\sin 30°$

$= 8.66 + 7.5 - 20$

$= -3.84$ KN

Resolve all the forces along Y axis,

$\Sigma F_y = 10\sin 30° + 15\sin 60° + 20 + 40\cos 30°$

$= 5 + 12.99 + 20 + 34.64$

$= 72.63$ KN.

Magnitude of resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-3.84)^2 + (72.63)^2}$

$= 72.73$ KN

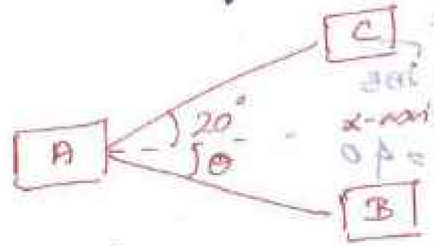
Direction of resultant, $\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

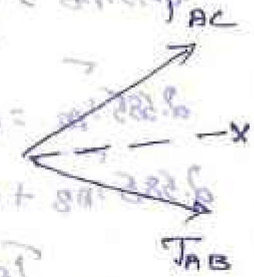
$\alpha = 86.97°$

A car A is pulled by means of two cars as shown in the fig. If the resultant of the forces acting on the car A is 40KN, being directed along the positive X axis, determine the angle θ of the cable attached to the car @ B, such that the force in cable AB is minimum. What is the magnitude of the force in each cable when it occurs?

Sol

For the force in cable AB to be minimum the angle between the forces should be equal to 90°

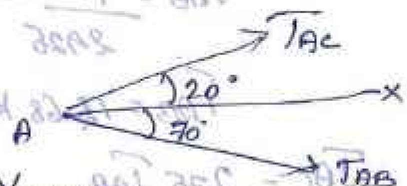




$\theta + 20° = 90°$

$\theta = 90° - 20°$

$= 70°$



Resolve all the forces along X axis.

$\Sigma F_x = 40$

$T_{AC}\cos 20° + T_{AB}\cos 70° = 40$

$0.94\,T_{AC} + 0.34\,T_{AB} = 40$

$0.94\,T_{AC} = 40 - 0.34\,T_{AB}$

$T_{AC} = \frac{40 - 0.34\,T_{AB}}{0.94} \longrightarrow (1).$

Resolve all the forces along y axis

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$\Sigma F_y = 0$

$T_{AC} \sin 20° - T_{AB} \sin 70° = 0$

$T_{AC} = \dfrac{T_{AB} \sin 70°}{\sin 20°}$

$T_{AC} = 2.75\, T_{AB} \longrightarrow (2)$

Sub (2) in (1)

$T_{AC} = \dfrac{40 - 0.34\, T_{AB}}{0.94}$

$2.75\, T_{AB} = \dfrac{40 - 0.34\, T_{AB}}{0.94}$

$2.585\, T_{AB} = 40 - 0.34\, T_{AB}$

$2.585\, T_{AB} + 0.34\, T_{AB} = 40$

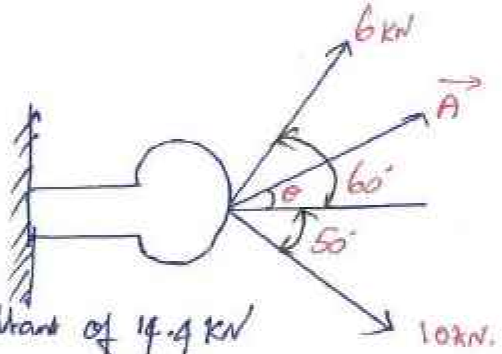
$T_{AB} = \dfrac{40}{2.925}$

$T_{AB} = 13.68$ KN

$T_{AC} = 2.75\, T_{AB}$
$= 2.75 \times 13.68$
$= 37.62$ KN.

Find the magnitude and direction θ of the force $\vec{A}$ so that the resultant of the system of forces shown in fig is horizontal and has a magnitude of 14.4 KN

Sol



Given that the resultant of 14.4 KN is horizontal, therefore $\Sigma F_x = 14.4$; $\Sigma F_y = 0$

Resolve all the forces along X-axis

$\Sigma F_x = A\cos\theta + 6\cos 60° + 10\cos 50° = 14.4$

$A\cos\theta + 3 + 6.43 = 14.4$

$A\cos\theta = 14.4 - 3 - 6.43$

$A\cos\theta = 4.97 \longrightarrow (1)$

$\Sigma F_y = 6\sin 60° + A\sin\theta - 10\sin 50° = 0$

$5.196 + A\sin\theta - 7.660 =$

$A\sin\theta = 7.660 - 5.196$

$A\sin\theta = 2.46 \longrightarrow (2)$

dividing eq (2) in (1)

$\tan\theta = \dfrac{2.46}{4.97} = \dfrac{A\sin\theta}{A\cos\theta}$

$\tan\theta = 0.49$

$\boxed{\theta = 26.3°}$

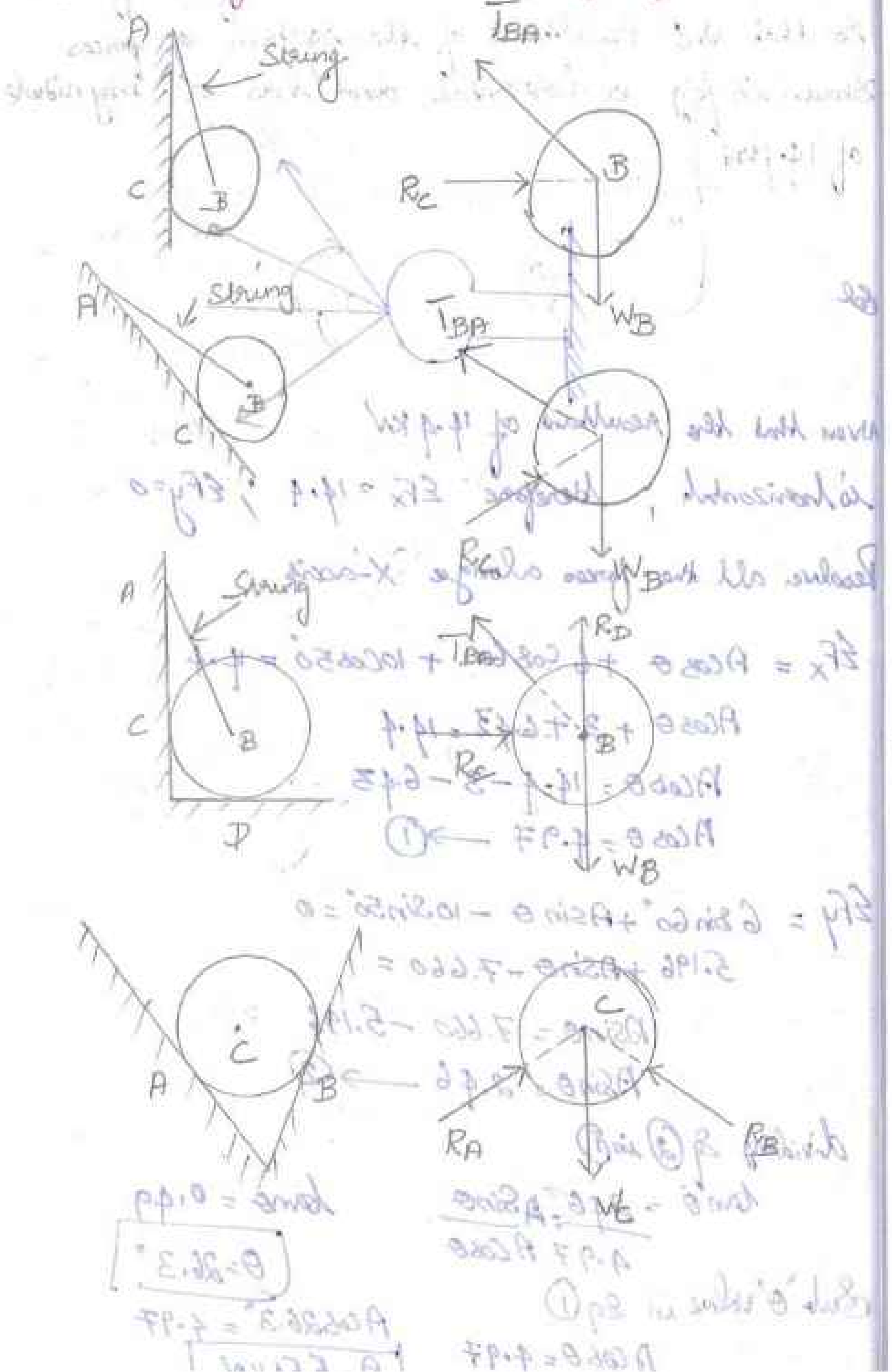
Sub θ value in eq (1)

$A\cos\theta = 4.97$

$A\cos 26.3° = 4.97$

Mechanical System

Free body diagram



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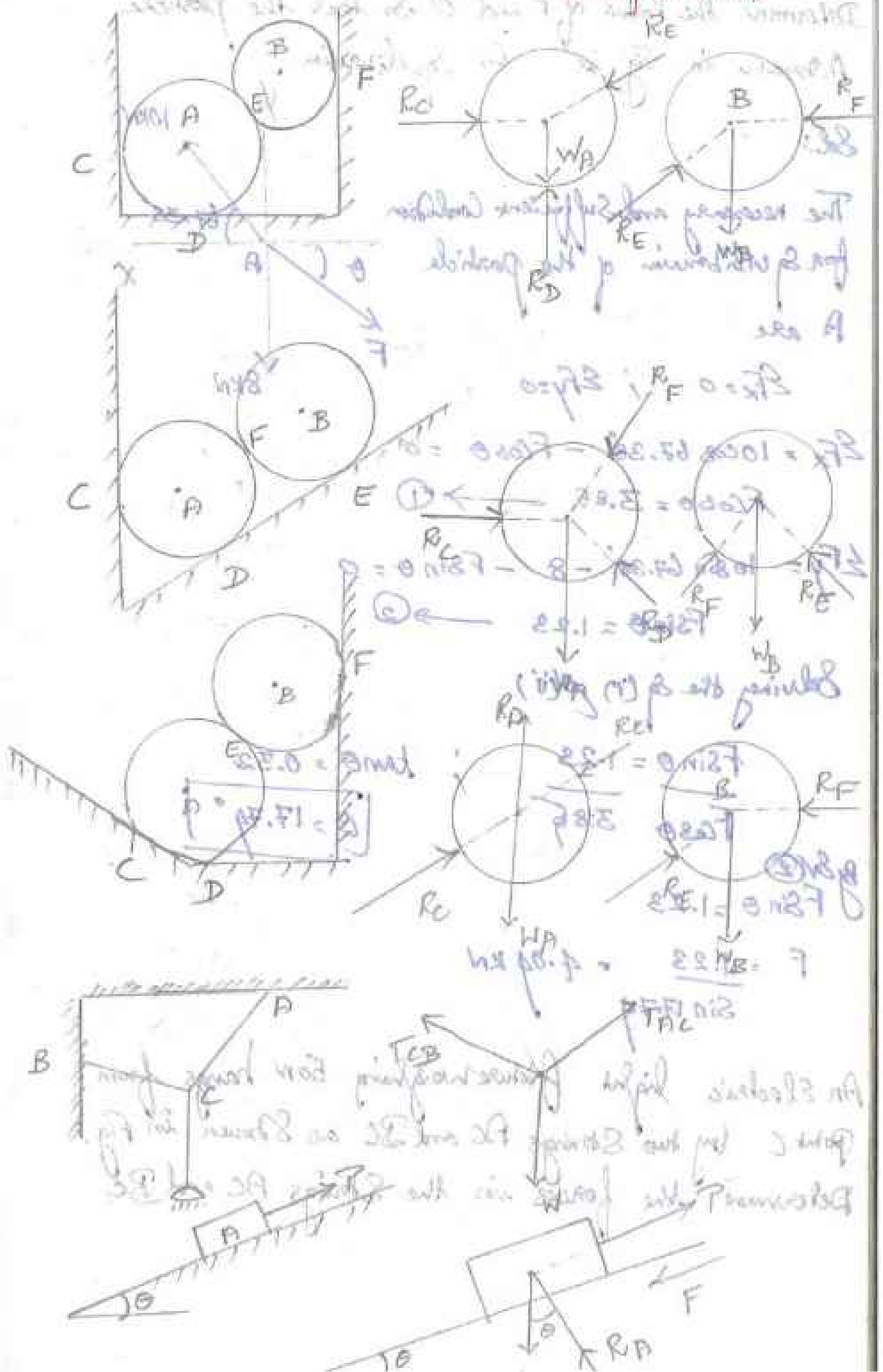
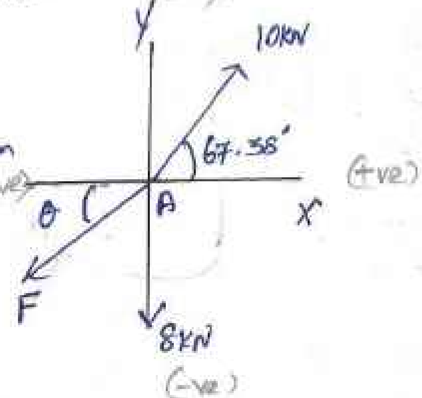


Determine the Values of F and θ so that the particle A shown in Fig is in Equilibrium



Sol:

The necessary and sufficient condition for Equilibrium of the particle A are

$\Sigma F_x = 0 \quad ; \quad \Sigma F_y = 0$

$\Sigma F_x = 10\cos 67.38° - F\cos\theta = 0$

$F\cos\theta = 3.85 \longrightarrow (1)$

$\Sigma F_y = 10\sin 67.38° - 8 - F\sin\theta = 0$

$F\sin\theta = 1.23 \longrightarrow (2)$

Solving the Eq (i) & (ii)

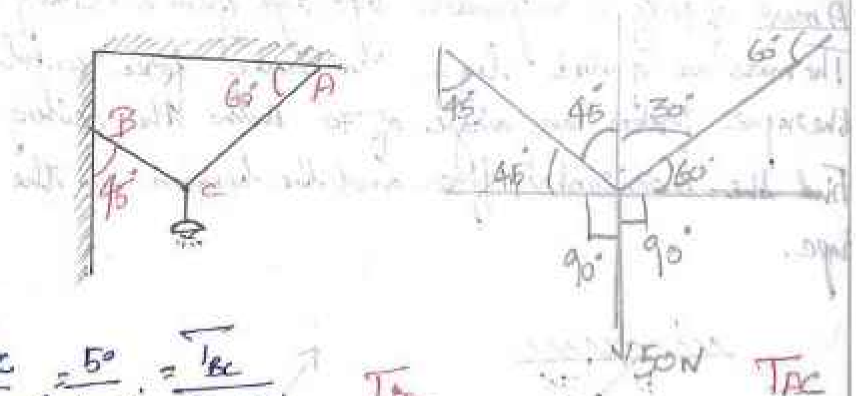
$\frac{F\sin\theta}{F\cos\theta} = \frac{1.23}{3.85} \quad ; \quad \tan\theta = 0.32 \quad \boxed{\theta = 17.74°}$

By Eq (2)

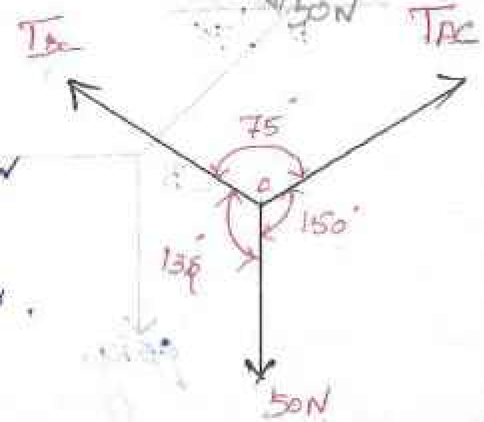
$F\sin\theta = 1.23$

$F = \frac{1.23}{\sin 17.74} = 4.04 \text{ KN}$

An Electric light fixture weighing 50N hangs from point C by two Strings AC and BC as shown in Fig. Determine the forces in the Strings AC and BC



$\frac{T_{AC}}{\sin 135} = \frac{50}{\sin 75} = \frac{T_{BC}}{\sin 150}$



$T_{AC} = \frac{50\sin 135}{\sin 75} = 36.60\text{ N}$

$T_{BC} = \frac{50\sin 150}{\sin 75} = 25.88\text{ N}$.

Alternative Solution:

$\Sigma F_x = 0 \quad T_{AC}\cos 60° - T_{BC}\cos 45 = 0$

$0.5T_{AC} - 0.707T_{BC} = 0 \longrightarrow (1)$

$\Sigma F_y = 0 \quad T_{AC}\sin 60° + T_{BC}\sin 45 - 50 = 0$

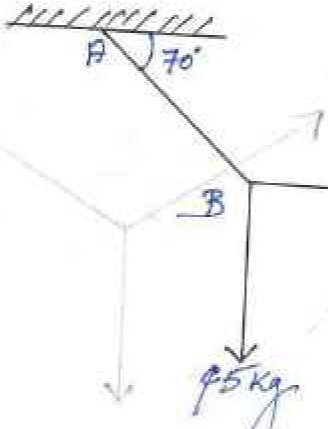
$0.866T_{AC} + 0.707T_{BC} = 50 \longrightarrow (2)$.

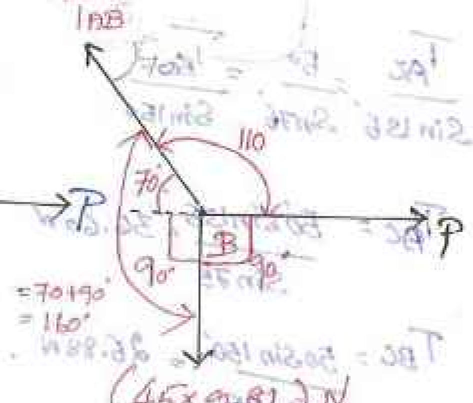
Solving the Eq (i) & (ii)

$T_{AC} = 36.60\text{ N}$

$T_{BC} = 25.88\text{ N}$.

A mass of 45 Kg is suspended by a rope from a ceiling. The mass is pulled by a horizontal force until the rope makes an angle of 70° with the ceiling. Find the horizontal force and the tension in the rope.





$(45 \times 9.81)\,N$

$= 441.45\,N$

By applying Lami's Equation @ B

$$\frac{T_{AB}}{\sin 90^\circ} = \frac{P}{\sin 160^\circ} = \frac{441.45}{\sin 110^\circ}$$

$$\frac{T_{AB}}{\sin 90^\circ} = \frac{441.45}{\sin 110^\circ}$$

$$T_{AB} = \frac{441.45 \times \sin 90^\circ}{\sin 110^\circ}$$

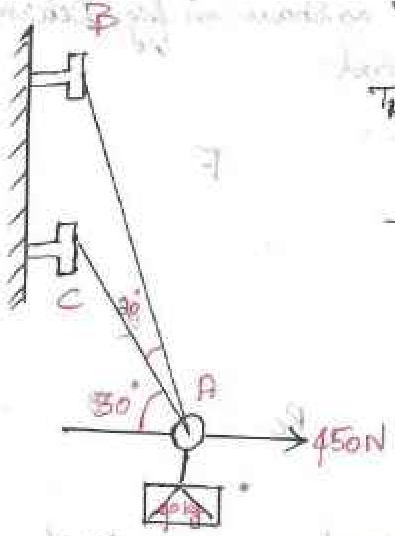
$$\boxed{T_{AB} = 469.78\,N}$$

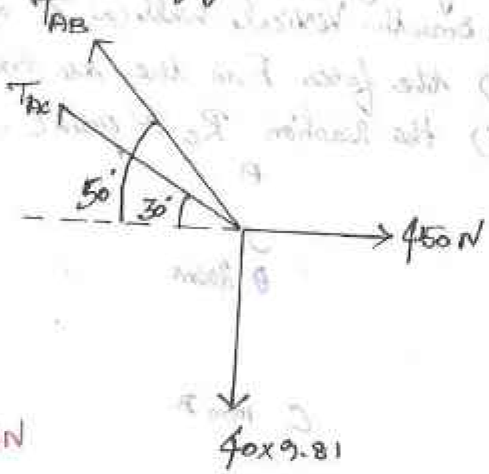
$$\frac{P}{\sin 160^\circ} = \frac{441.45}{\sin 110^\circ}$$

$$P = \frac{441.45 \times \sin 160^\circ}{\sin 110^\circ}$$

$$\boxed{P = 160.67\,N}$$

Determine the tension in cables AB and AC required to hold the 40 kg crate shown in fig.





40×9.81

$= 392.4\,N$

Sol:

The joint A is in Equilibrium. The free body diagram of the joint A is shown in fig. Let T_{AB} and T_{AC} are the tension in cables AB and AC respectively.

For $\Sigma F_x = 0$

$$450 - T_{AB}\cos 50 - T_{AC}\cos 30 = 0$$

$$0.642\,T_{AB} + 0.866\,T_{AC} = 450 \longrightarrow (1)$$

For $\Sigma F_y = 0$

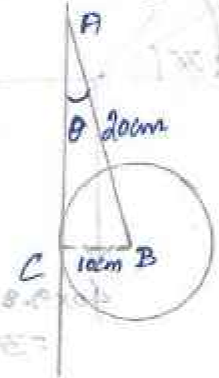
$$T_{AB}\sin 50^\circ + T_{AC}\sin 30 - 392.4 = 0$$

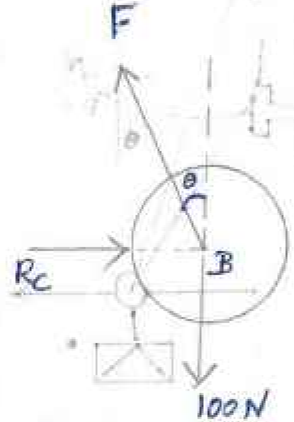
$$0.766\,T_{AB} + 0.5\,T_{AC} = 392.4 \longrightarrow (2)$$

Solving the Eq (i) & (ii)

$$\boxed{\begin{aligned} T_{AB} &= 335.38\,N \\ T_{AC} &= 271\,N. \end{aligned}}$$

A circular roller of weight 100 N and radius 10 cm hangs by a tie rod AB = 20 cm and rests against a smooth vertical wall @ C as shown in fig. Determine
i) the force F in the tie rod
ii) the reaction R_C @ point C.





Sol:
Given, weight of roller, W = 100N
Radius of roller, BC = 10cm
Length of tie rod, AB = 20cm

R_C = Reaction @ C
F = Force in the tie rod AB

From ΔABC, $\sin\theta = \frac{BC}{AB} = \frac{10}{20} = 0.5$

$\theta = \sin^{-1}(0.5) = 30°$

For Equilibrium of the roller,
$\Sigma F_x = 0$ & $\Sigma F_y = 0$

for $\Sigma F_x = 0$

$R_C - F\sin\theta = 0$

$R_C = F\sin 30 \rightarrow (1)$

For $\Sigma F_y = 0$

$F\cos\theta - 100 = 0$

$100 = F\cos 30 \rightarrow (2)$

$F = \frac{100}{\cos 30}$

$\boxed{F = 115.47 N}$

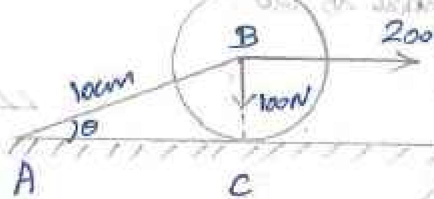
Sub F value in eq (1)

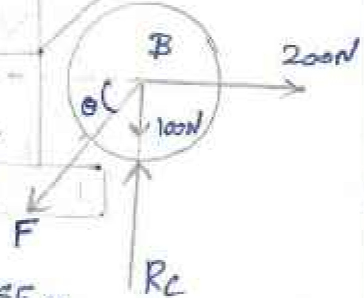
$R_C = F\sin 30$
$= 115.47 \sin 30°$

$\boxed{R_C = 57.73 N}$

A circular roller of radius 5cm and of weight 100N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 10cm as shown in fig. A horizontal force of 200N is acting @ B. Find the tension in the bar AB and the vertical reaction @ C.

Sol:
Given: Weight, W = 100N
Radius, BC = 5cm
Length of bar, AB = 10cm
Horizontal force @B = 200N





From Δ ABC, $\sin\theta = \frac{BC}{AB} = \frac{5}{10} = 0.5$

$\theta = \sin^{-1}(0.5) = 30°$

Let T - Tension in string AB

For Equilibrium of the roller $\Sigma F_x = 0$ & $\Sigma F_y = 0$

for $\Sigma F_x = 0$

$200 - F\cos 30° = 0$

$F\cos 30° = 200$

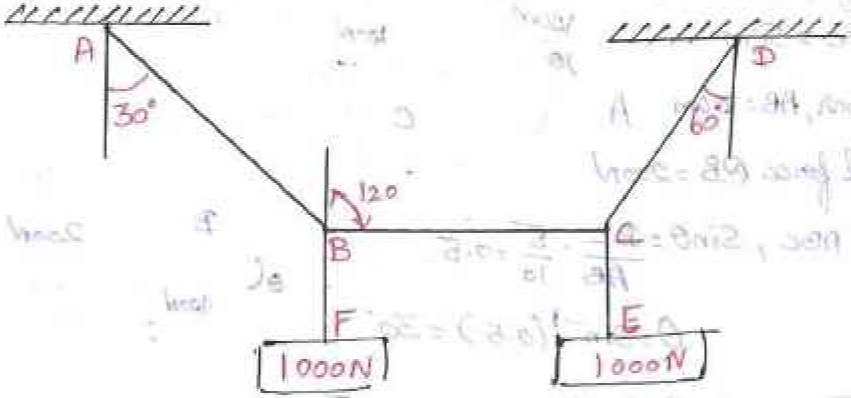
$F = \frac{200}{\cos 30°} = 230.94 N$

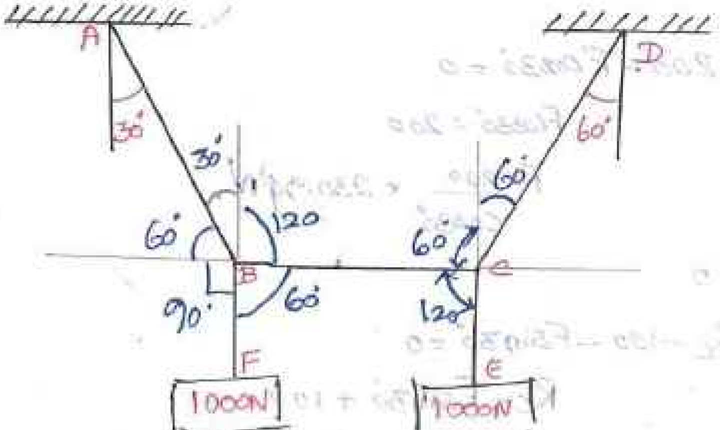
for $\Sigma F_y = 0$

$R_C - 100 - F\sin 30° = 0$

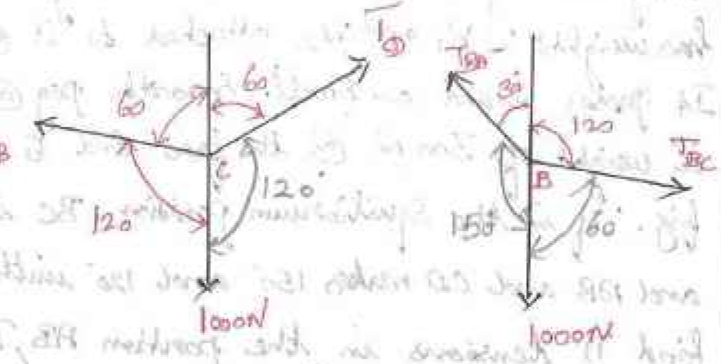
$R_C = F\sin 30° + 100$
$= 230.94 \sin 30° + 100$
$= 115.47 + 100$
$R_C = 215.47 N$

A String ABCD, attached to two fixed points A and D has two equal weights of 1000N attached to it @ B and C. The weights rest with the portions AB and CD inclined @ angles of 30° and 60° respectively, to the Vertical as shown in fig. Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the Vertical is 120°.





Dr. G. PUGAZHMANI



Sol.

For the sake of convenience, let us split up the string ABCD into two parts. The system of forces @ joints B and C is shown in fig.

Let T_{AB} and T_{BA} are the tensions in the string AB, acting from A to B and B to A respectively.

For the equilibrium of string AB,

$$T_{AB} = T_{BA}$$

Similarly $T_{BC} = T_{CB}$ and $T_{CD} = T_{DC}$

Applying Lami's Equation @ point C,

$$\frac{T_{CD}}{\sin 120} = \frac{1000}{\sin 120} = \frac{T_{BC}}{\sin 120°}$$

$$\frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°} \qquad \frac{T_{BC}}{\sin 120} = \frac{1000}{\sin 120°}$$

$$\boxed{T_{CD} = 1000 \times \frac{\sin 120°}{\sin 120} = 1000N} \qquad \boxed{T_{BC} = 1000 \times \frac{\sin 120}{\sin 120°} = 1000N}$$

Lami's @B,

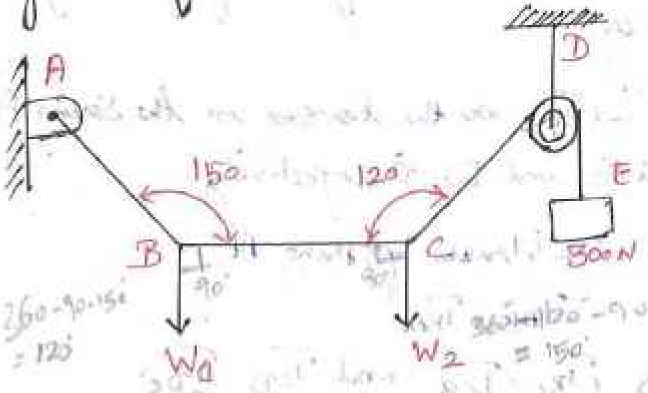
$$\frac{T_{AB}}{\sin 60°} = \frac{1000}{\sin 150} = \frac{T_{BC}}{\sin 150°}$$

$$\frac{T_{AB}}{\sin 60°} = \frac{1000}{\sin 150°}$$

$$\boxed{T_{AB} = 1732.0\,N}$$

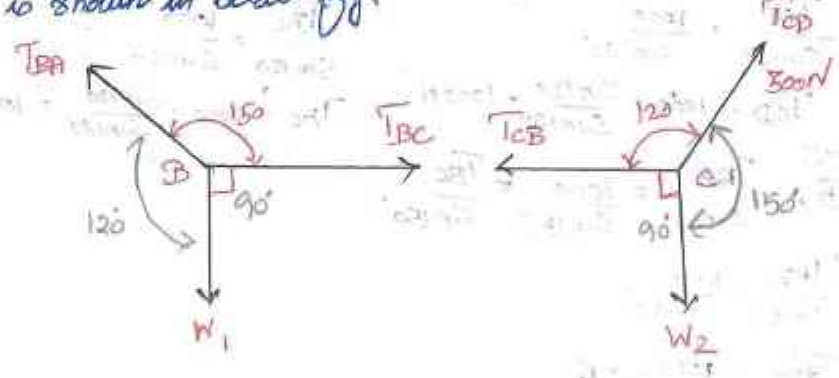
A light string ABCDE whose extremity A is fixed, has weights W_1 and W_2 attached to it @ B and C. It passes round a small smooth peg @ D carrying a weight of 300 N @ the free end E as shown in fig. If in the equilibrium position, BC is horizontal and AB and CD makes 150° and 120° with BC, find i) tensions in the portion AB, BC and CD of the string

ii) Magnitudes of W_1 and W_2.



Sol: Given, weight @ E = 300 N

For the sake of convenience, let us split up the string ABCD into two parts. The system of forces @ joint B and C is shown in below fig.



(i) Tensions in the portion AB, BC and CD of the string

let T_{AB} = Tension in the portion AB

T_{BC} = Tension in the portion BC

we know that tension in the portion CD of the string

$$T_{CD} = T_{DE} = 300 N.$$

Applying Lami's Equation @ C,

$$\frac{T_{CB}}{\sin 150°} = \frac{W_2}{\sin 120°} = \frac{300}{\sin 90°}$$

$$T_{CB} = 300 \times \frac{\sin 150}{\sin 90}$$

$$\boxed{T_{CB} = 150 N}$$

$$\frac{300}{\sin 90°} = \frac{W_2}{\sin 120°}$$

$$W_2 = 300 \times \frac{\sin 120}{\sin 90°}$$

$$\boxed{W_2 = 259.81 N}$$

Again applying Lami's Equation @ B

$$\frac{T_{AB}}{\sin 90°} = \frac{W_1}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

150 N

$$\frac{T_{AB}}{\sin 90°} = \frac{150}{\sin 120°}$$

$$T_{AB} = 150 \times \frac{\sin 90°}{\sin 120°}$$

$$\boxed{T_{AB} = 173.20 N}$$

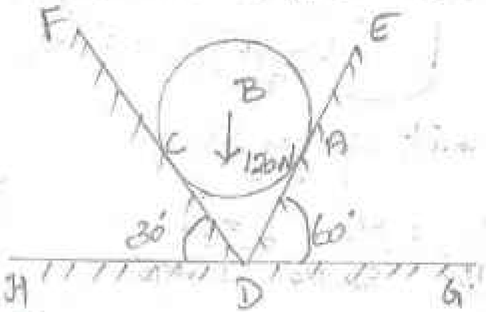
$$\frac{W_1}{\sin 150°} = \frac{150}{\sin 120°}$$

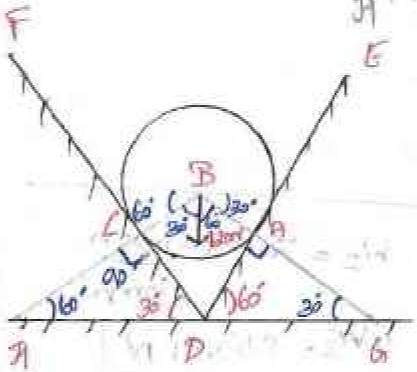
$$W_1 = 150 \times \frac{\sin 150°}{\sin 120°}$$

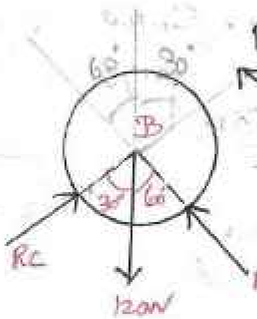
$$\boxed{W_1 = 86.60 N}$$

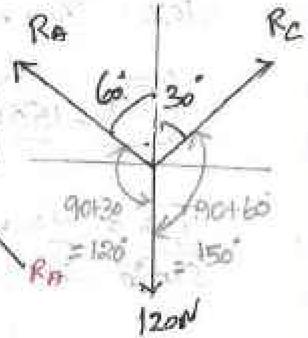
Sides of Angles Triangle $= (2n-4)90$

A ball of weight 120 N rests in a right angled groove, as shown in fig. The sides of the groove are inclined to an angle of 30° and 60° to the horizontal. If all the surfaces are smooth, then determine the reaction R_A and R_C @ the point of contact.









Sol: Given,

Weight of ball, $W = 120N$; Angle of groove $= 90$

Angle $FDH = 30°$; Angle $EDG = 60°$

The forces acting on the ball are

1. Weight of the ball $= 120N$ and acting vertically downwards
2. Reaction @ C, R_C acting normal to FD
3. Reaction @ A, R_A acting normal to DG

Applying Lami's Equation @ B,

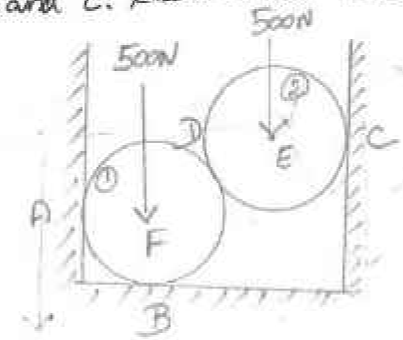
$$\frac{R_A}{\ } = \frac{120}{\ } = \frac{R_C}{\ }$$

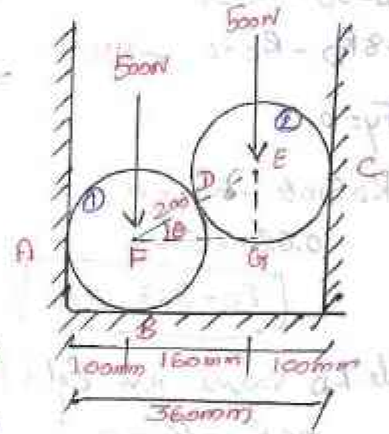
Dr. Gopinath

$$\therefore R_B = \frac{120}{\sin 90°} \times \sin 150° = 60N$$

$$\therefore R_C = \frac{120}{\sin 90°} \times \sin 120° = 103.92N.$$

Two spheres each of weight 500N and of radius 100 mm rest in a horizontal channel of width of 360mm as shown in fig. Find the reactions on the point of contact A, B and C. Assume all the surfaces to be smooth.





Sol. Given

Weight of each sphere, $W = 500N$

Radius of each sphere, $R = 100N$

Width of horizontal channel $= 360mm$

Join the center E to center F

Now $EF = 100 + 100 = 200\,mm$, $FG = 160mm$

$\triangle EFG$,

$$EF^2 = FG^2 + GE^2$$

$$EG = \sqrt{EF^2 - FG^2} = \sqrt{200^2 - 160^2} = 120mm$$

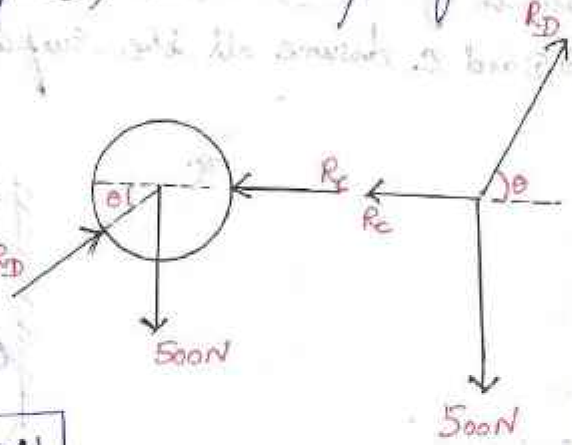
$$\therefore \sin\theta = \frac{opp}{hyp} = \frac{GE}{EF} = \frac{120}{200} = 0.6 \qquad \cos\theta = \frac{adj}{hyp} = \frac{FG}{FE} = \frac{160}{200} = 0.8$$

← PTO $\cos\theta = 0.8$, $\sin\theta = 0.6$

Equilibrium of Sphere (2)

The sphere (2) has point of contact @ C and D. hence the forces acting on sphere (2) are

* weight 500N, acting vertically downward
* Reaction R_C at point C, normal to the wall surface
* Reaction R_D at point D, normal to the point of contact.



For $\Sigma F_x = 0$

$R_D\cos\theta - R_C = 0$

$0.8R_D - R_C = 0 \rightarrow (i)$

For $\Sigma F_y = 0$

$R_D\sin\theta - 500 = 0$

$0.6R_D = 500$

$\boxed{R_D = 833.33N}$

Sub R_D value in Eq(i)

$0.8R_D - R_C = 0$

$0.8 \times (833.33) - R_C = 0$

$R_C = 0.8 \times 833.3$

$\boxed{R_C = 666.66N}$

Equilibrium of Sphere (1)

The sphere (1) has points of Contact @ A, B and D.

hence the forces acting on sphere (1) are

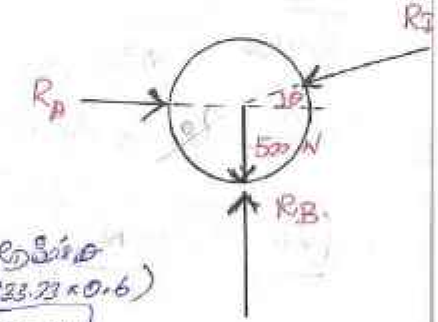
* weight 500N, acting vertically downward
* Reaction R_A @ point A, normal to wall surface
* Reaction R_B @ point B, Normal to the base

$\Sigma F_x = 0$

$R_A - R_D\cos\theta = 0$

$R_A = R_D\cos\theta = (833.33 \times 0.8)$

$\boxed{R_A = 666.66N}$



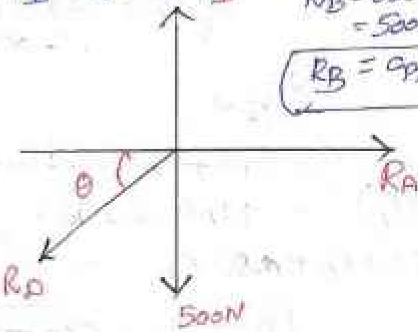
$\Sigma F_y = 0$

$R_B - R_D\cos\theta - 500 = 0$

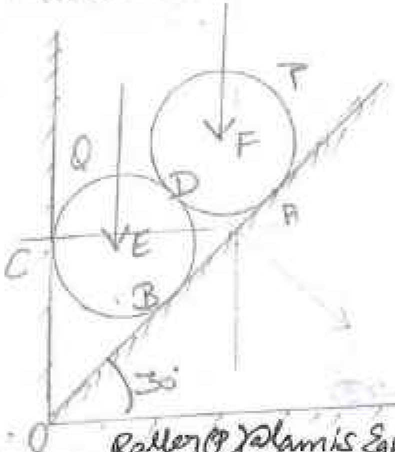
$R_B = 500 + R_D\sin\theta$

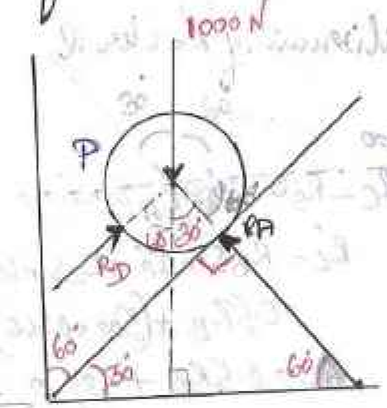
$= 500 + (833.33 \times 0.6)$

$\boxed{R_B = 999.98N}$



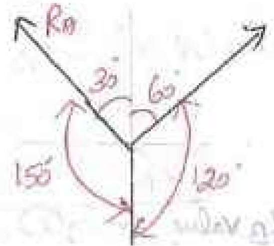
Two identical rollers, each of weight W = 1000N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions @ the points of supports A, B and C. Assume all the surfaces to be smooth.





Roller (P) @ Lami's Equation

$\frac{R_A}{\sin 120°} = \frac{1000}{\sin 90°} = \frac{R_D}{\sin 150°}$

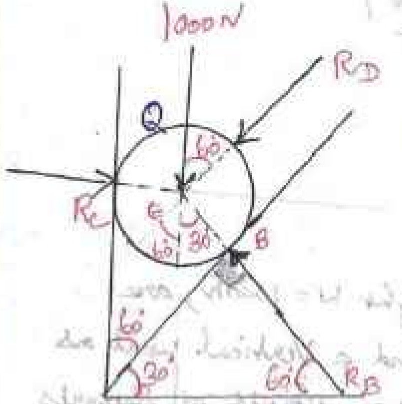


$R_A = \frac{1000 \times \sin 120°}{\sin 90°} = 866.0N$

$$\frac{R_D}{\sin 150} = \frac{1000}{\sin 90^\circ}$$

$$R_D = 1000 \times \frac{\sin 150^\circ}{\sin 90^\circ}$$

$$\boxed{R_D = 500\ N}$$



Equilibrium of Roller Q

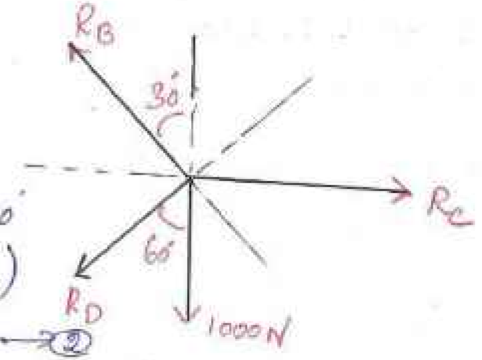
$\Sigma F_x = 0$

$R_C - R_B \sin 30^\circ - R_D \sin 60^\circ = 0$

$R_C = R_B \sin 30^\circ + R_D \sin 60^\circ$

$= 0.5 R_B + (500 \times 0.866)$

$R_C = 0.5 R_B + 433.0 \quad \rightarrow (2)$



$\Sigma F_y = 0 \quad R_B \cos 30^\circ - R_D \cos 60^\circ - 1000 = 0$

$R_B \cos 30^\circ = 1000 + 500(\cos 60^\circ)$

$$\boxed{R_B = 1443.37}$$

Sub R_B value in Eq (2)

By alternative solution

Equilibrium of Roller P

$\Sigma F_x = 0$

$R_D \sin 60^\circ - R_A \sin 30^\circ = 0$

$R_D \sin 60^\circ = R_A \sin 30^\circ$

$$\boxed{R_D = 0.577 R_A} \rightarrow (1)$$

$\Sigma F_y = 0$

$R_D \cos 60^\circ + R_B \cos 30 - 1000 = 0$

$(0.577 R_A) \cos 60^\circ + R_A \cos 30^\circ = 1000$

$0.288 R_A + 0.866 R_A = 1000$

$R_A = \frac{1000}{1.154} = \boxed{866.17\ N}$

Sub R_A in Eq (1)

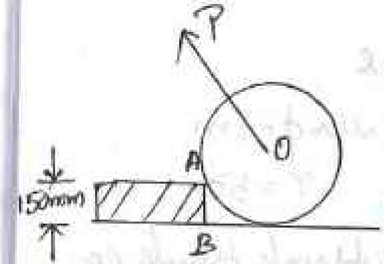
$R_D = 0.577 R_A$

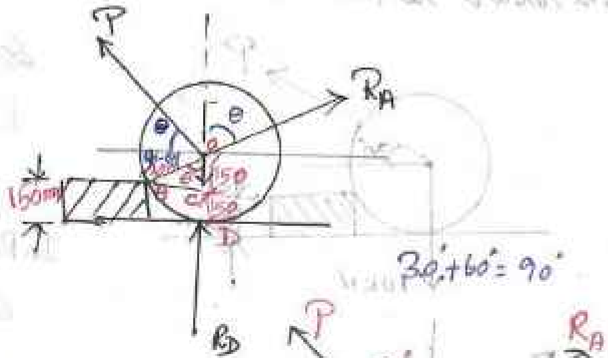
$= 0.577 \times 866.17$

$$\boxed{R_D = 500\ N}$$

Dr. Lognanathamani

A uniform wheel 600 mm in diameter rests against a rigid rectangular block 150 mm thick as shown in fig. Find the least pull P, through the center of the wheel in order to just turn the wheel over the corner of the block. All surfaces are smooth. Find also the reaction of the block. The wheel weight 900N.



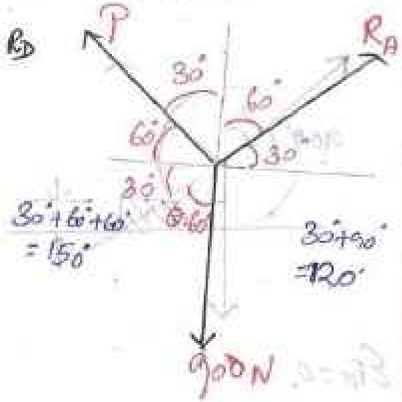


$30^\circ + 60^\circ = 90^\circ$

In ΔAOC

$\cos\theta = \frac{OC}{OA} = \frac{150}{300} = \frac{1}{2}$

$$\boxed{\theta = 60^\circ}$$



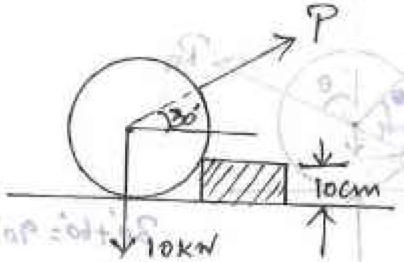
Using Lami's theorem

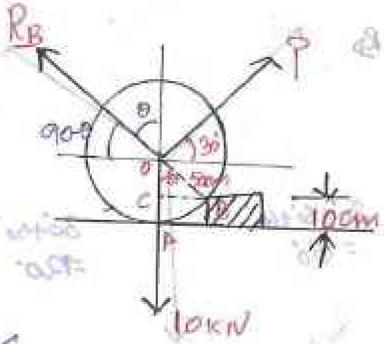
$$\frac{P}{\sin 120^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{900}{\sin 90^\circ}$$

$$P = 900 \times \frac{\sin 120^\circ}{\sin 90^\circ} = 779.42\ N$$

$$R_A = 900 \times \frac{\sin 150^\circ}{\sin 90^\circ} = 450\ N.$$

A cylindrical roller has a weight of 10kN and it is being pulled by a force is inclined @ 30° with the horizontal as shown in fig. While moving it comes across an obstacle 10cm high. Calculate the force required to cross this obstacle, if the diameter of the roller is 1m.





Sol

Roller $\phi = 1m$

$r = 50cm$

In Right angle triangle OBC,

OB = Radius = 50cm

$OA = 50cm = \frac{1m}{2}$

$OC = OA - CA$

$= 50 - 10$

$OC = 40cm$ ← Box thickness

$\cos\theta = \frac{OC}{OB} = \frac{40}{50}$

$\theta = \cos^{-1}(4/5) = 36.87°$

$\Sigma F_x = 0$

$P\cos 30 - R_B \cos 53.13 = 0$

$P\cos 30 = R_B \cos 53.13$ $\quad (90 - \theta = 53.13)$

$P = 0.693 R_B \longrightarrow (1)$

$\Sigma F_y = 0$

$P\sin 30° + R_B \sin 53.13° - 10 = 0$

$0.693 R_B \sin 30° + R_B \sin 53.13° - 10 = 0$

$0.3465 R_B + 0.8 R_B = 10$

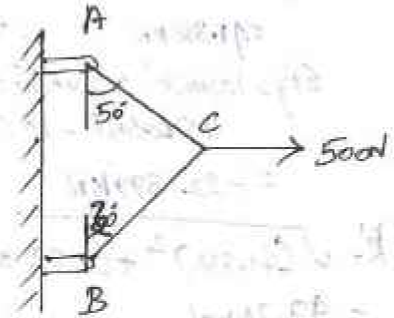
$\boxed{R_B = 8.721 kN}$

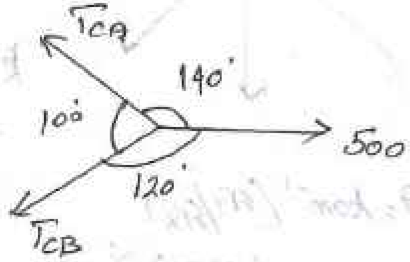
Sub R_B in (1)

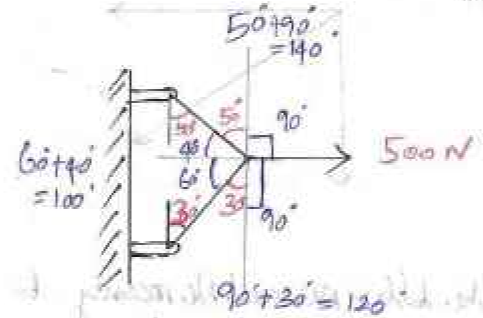
$P = 0.693 R_B$

$\boxed{P = 6.04 kN}$

Two cables are tied together @ C and are loaded as shown in fig. Determine the tension in the cable AC and BC.







Sol

let T_{CB} = Tension in the String CB from C to B

T_{CA} = Tension in the String CA from C to A

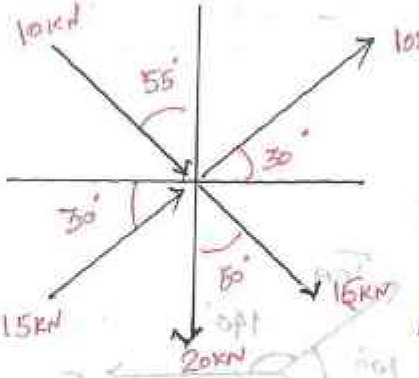
Applying Lami's Equation @ C.

$$\frac{T_{CA}}{\sin 120°} = \frac{T_{CB}}{\sin 140°} = \frac{500}{\sin 100°}$$

$\therefore T_{CA} = \frac{500}{\sin 100} \times \sin 120° = \boxed{439.69 N}$

$\therefore T_{CB} = \frac{500}{\sin 100°} \times \sin 140° = \boxed{326.35 N}$

Determine the magnitude and direction of the resultant force acting @ O



$\Sigma F_x = 10\cos 30° + 15\cos 30° + 15\sin 50° + 10\sin 55$

$= 41.332\,kN$

$\Sigma F_y = 10\sin 30° + 15\sin 30° - 20 - 15\cos 50° - 10\cos 55$

$= -22.899\,kN$

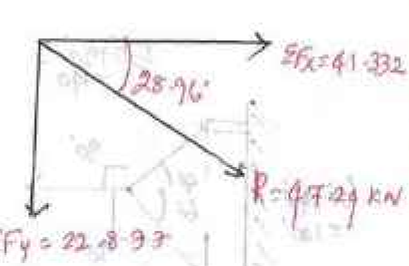
$R = \sqrt{(41.332)^2 + (-22.899)^2}$

$= 47.24\,kN$

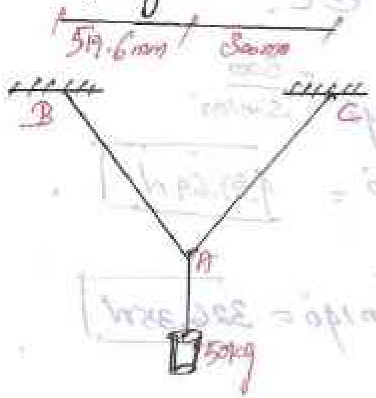
$\theta = \tan^{-1}(\Sigma F_y / \Sigma F_x)$

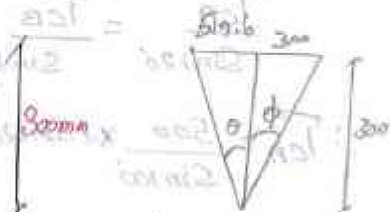
$= \tan^{-1}\left(\frac{22.899}{41.332}\right)$

$\theta = 28.96°$



Determine the tension in the cables AB and AC necessary to support the 50kg cylinder as shown in fig.





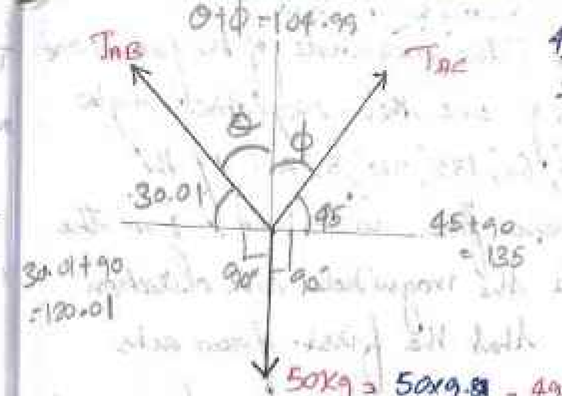
$\tan\theta = \frac{519.6}{300}$

$\theta = 59.99°$

$\tan\phi = \frac{300}{300}$

$\phi = 45°$

$\theta + \phi = 104.99$



$50kg = 50 \times 9.81 = 490.5\,N$

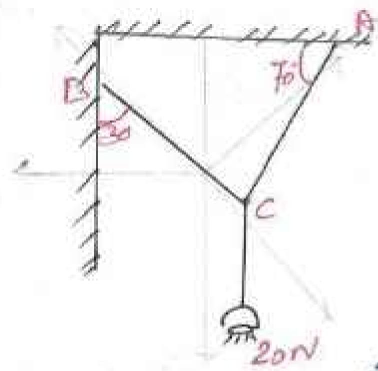
$\frac{490.5}{\sin 104.99} = \frac{T_{AB}}{\sin 135} = \frac{T_{AC}}{\sin 120.01}$

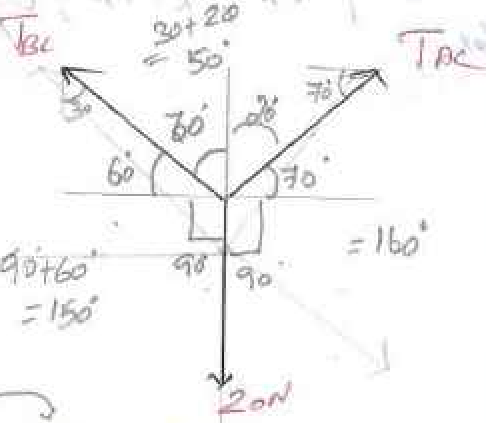
$\boxed{T_{AB} = 359.01\,N}$

$\boxed{T_{AC} = 439.70\,N}$

$1kg = 9.81\,N$

An Electric light fixture weighing 20N hangs from a point C by two strings AC and BC. AC is inclined @ 70° to horizontal and BC @ 30° to vertical. Using Lami's theorem determine the force in string AC and BC.





$\frac{20}{\sin 50°} = \frac{T_{AC}}{\sin 150°} = \frac{T_{BC}}{\sin 160°}$

$T_{AC} = 20 \times \frac{\sin 150°}{\sin 50°} = 13.05\,N$

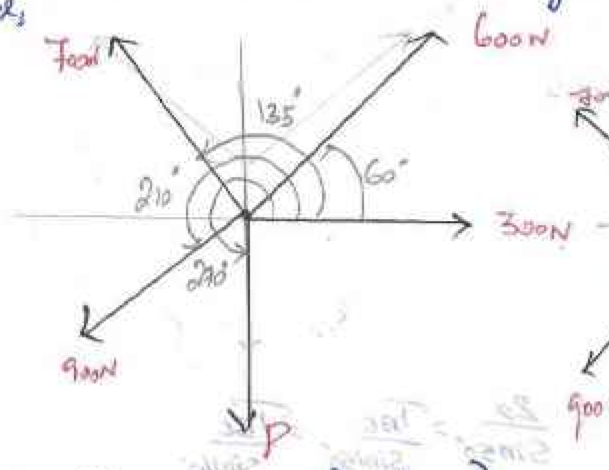
$T_{BC} = 20 \times \frac{\sin 160°}{\sin 50} = 8.92\,N$

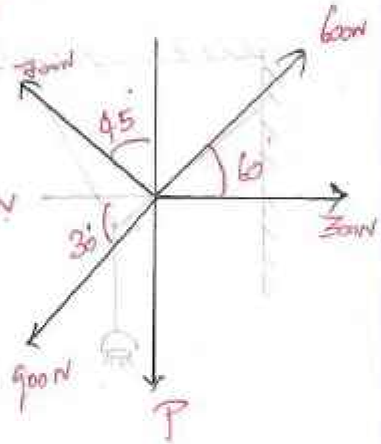
Five forces acting on a particle, the magnitude of the forces are 300N, 600N, 700N, 900N and P and their respective angles with the horizontal are 0°, 60°, 135°, 210° & 270°. If the vertical component of all these forces is −1000N! Find the value of P and also calculate the magnitude and direction of the resultant, assuming that the first force acts towards the point while all the remaining forces acts away from the point. (actaway from a point)

Given: $\Sigma F_y = -1000N$

To find: $P = ?N$; $R = ?N$; $\theta = ?$ degrees

Sol;





$\Sigma F_x = 0$ (forces along x-direction)

$$300 + 600 \times \cos 60° - 700 \sin 45 - 900 \cos 30° = \Sigma F_x$$

$$300 + 300 - 494.97 - 779.4 = \Sigma F_x$$

$$\boxed{\Sigma F_x = -674.37\ N}$$

Given Data.

Forces along y-direction ($\Sigma F_y = 0$) $\Sigma F_y = -1000\ N$

$$\Sigma F_y = 600 \sin 60 + 700 \cos 45 - 900 \sin 30° - P$$

Dr. G.[illegible]

$$-1000 = 519.61 + 494.97 - 450 - P$$

$$P = 519.61 + 494.97 - 450 + 1000$$

$$\boxed{P = 1564.58\ N}$$

Resultant of forces

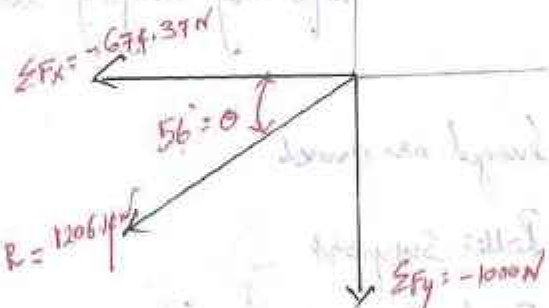
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(674.37)^2 + (1000)^2} = 1206.14\ N$$

$$\theta = \tan^{-1}(\Sigma F_y / \Sigma F_x)$$

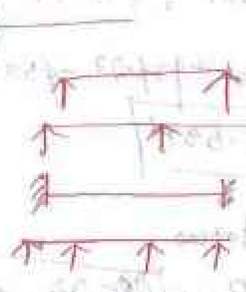
$$= \tan^{-1}\left(\frac{1000}{674.37}\right)$$

$$\theta = 56.00°$$



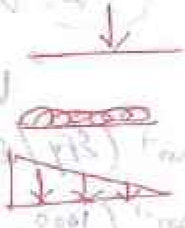
Types of Beam.

* Cantilever Beam
* Simply Supported Beam
* Over hanging Beam
* Fixed Beam
* Continous Beam



Types of Load :-

* Concentrated or point load
* Uniformly distributed load
* Uniformly Varying load.



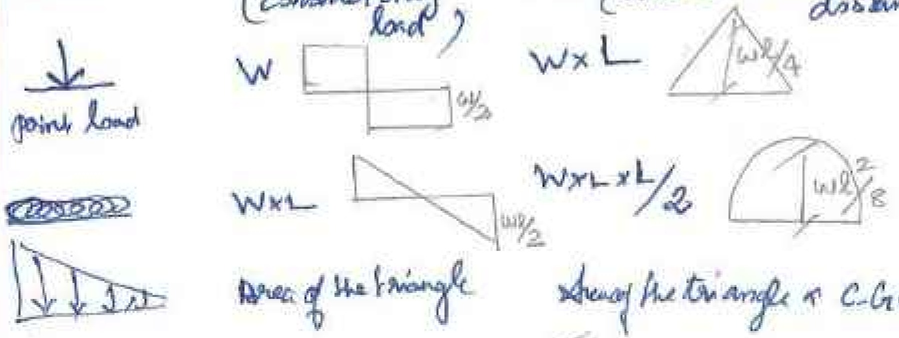
Support :-

hinged or pinned

Roller Support

Fixed Support

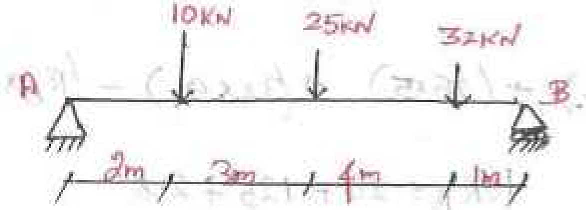
load	Shear Force (SF) (consider only load)	Bending moment. (BMD) (consider load & its distance)
point load	W	W × L
	W × L	W×L×L/2
	Area of the triangle	Area of the triangle × C.G distance



CG

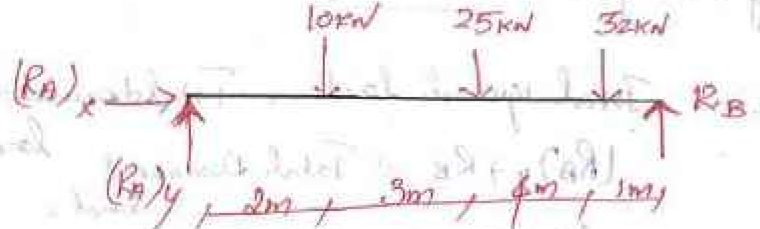
$(0.66 l) = \frac{2}{3} l$ = @ zero pts

Find the reaction @ support A and B of the beam shown in fig



Sol:-

The free body diagram of beam is drawn. The reaction components of hinged and roller supports are shown in fig. In roller support, only one reaction, R_B acting normal to the plane of rollers. In hinged support, there are two reaction component $(R_A)_x$ and $(R_A)_y$.



For Equilibrium of beam -

$\Sigma F_x = 0$ ($\rightarrow$ +ve $\leftarrow$ -ve)

$(R_A)_x = 0$

$\Sigma F_y = 0$ ($\uparrow$ +ve $\downarrow$ -ve)

$$(R_A)_y - 10 - 25 - 32 + R_B = 0$$

$$(R_A)_y + R_B = 10 + 25 + 32$$

$$\boxed{(R_A)_y + R_B = 67 \text{ KN}}$$

$\Sigma M_A = 0$ (↺ +ve, ↻ −ve)

$$(10\times 2) + (25\times 5) + (32\times 9) - (R_B \times 10) = 0$$

$$10R_B = 20 + 125 + 288$$

$$10R_B = 433$$

$$R_B = 433/10$$

$$\boxed{R_B = 43.3\,kN}$$

$$(R_A)_y + R_B = 67$$

$$(R_A)_y + 43.3 = 67$$

$$(R_A)_y = 67 - 43.3 = 23.7\,kN$$

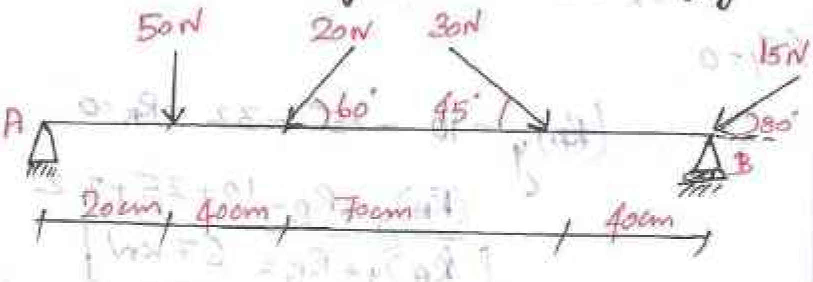
<u>To check:</u> Total upward load = Total downward load

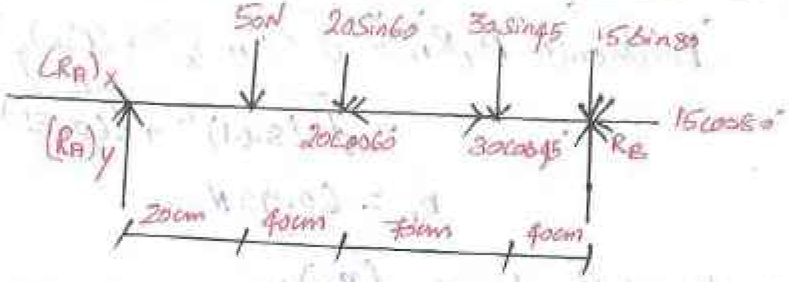
$(R_A)_y + R_B$ = Total Downward load.

$$43.3 + 23.7 = 10 + 25 + 32$$

$$67\,kN = 67\,kN.$$

Q) Find the Support reaction of the beam shown in fig.





For Equilibrium of beam,

$\Sigma F_x = 0$ (← −ve, → +ve)

$$(R_A)_x - 20\cos 60° + 30\cos 45° - 15\cos 80° = 0$$

$$(R_A)_x = 20\cos 60° - 30\cos 45° + 15\cos 80°$$

$$= 10 - 21.21 + 2.60$$

$$\boxed{(R_A)_x = -8.61\,N}$$

$\Sigma F_y = 0$

$$(R_A)_y - 50 - 20\sin 60° - 30\sin 45° - 15\sin 80° + R_B = 0$$

$$(R_A)_y + R_B = 50 + 20\sin 60° + 30\sin 45° + 15\sin 80°$$

$$(R_A)_y + R_B = 50 + 17.32 + 21.21 + 14.77$$

$$\boxed{(R_A)_y + R_B = 103.3}$$

$\Sigma M_A = 0$

$$(50\times 20) + (20\sin 60° \times \underset{20+40}{60}) + (30\sin 45° \times \underset{20+40+70}{130}) + (15\sin 80° \times \underset{20+40+70+40}{170}) - (R_B \times 170) = 0$$

$$1000 + 1039.23 + 2757.71 + 2511.26 = 170R_B$$

$$170R_B = 7308.199$$

$$\boxed{R_B = 42.98\,N}$$

$$(R_A)_y + R_B = 103.3$$

$$(R_A)_y = 103.3 - 42.98 = 60.32\,N$$

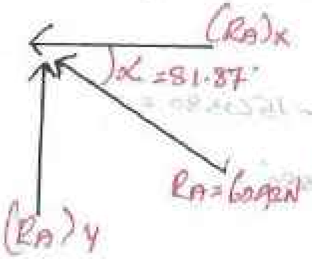
Reaction @ A, $R_A = \sqrt{(R_A)_x^2 + (R_A)_y^2}$

$$= \sqrt{(8.61)^2 + (60.31)^2}$$

$$\boxed{R_A = 60.92 N}$$

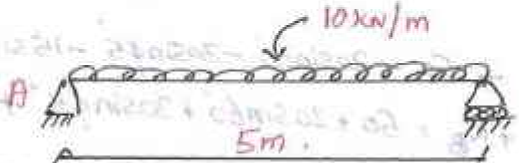
Direction of R_A, $\tan\alpha = \dfrac{(R_A)_y}{(R_A)_x}$

$$= \frac{60.31}{8.61} = 7.00$$



$$\boxed{\alpha = 81.87'}$$

Q) Find the reaction @ point A and B for the beam shown in fig.



$\Sigma M_B = 0$ ↻ +ve

$$(R_A \times 5) - 10 \times 5 \times 5/2 = 0$$

$$5R_A = 125$$

$$\boxed{R_A = 25 KN}$$

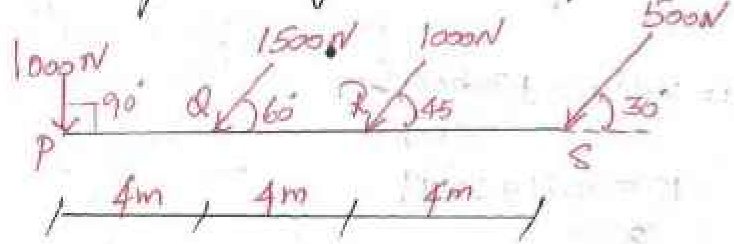
Total upward load = Total Downward load

$$R_A + R_B = 10 \times 5$$

$$R_B = (10 \times 5) - 25$$

$$\boxed{R_B = 25 KN}$$

A horizontal line PQRS is 12m long, where PQ = QR = RS = 4m. Forces of 1000N, 1500N, 1000N and 500N act @ P, Q, R and S respectively with downward direction. The lines of action of these forces makes angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant forces.



↑ +ve

$$\Sigma H = -1000\cos 90° - 1500\cos 60° - 1000\cos 45° - 500\cos 30°$$

$$= (-1000 \times 0) - (1500 \times 0.5) - (1000 \times 0.707) - (500 \times 0.866)$$

$$= -0 - 750 - 707 - 433$$

$$= -1890 N$$

+ve

$$\Sigma V = -1000\sin 90° - 1500\sin 60° - 1000\sin 45° - 500\sin 30°$$

$$= (-1000 \times 1) - (1500 \times 0.866) - (1000 \times 0.707) - (500 \times 0.5)$$

$$= -1000 - 1299 - 707 - 250$$

$$= -3256 N$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-1890)^2 + (-3256)^2}$$

$$R = 3764.7 \approx 3765 N.$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{3256}{1890}\right) = 59.8°$$

Let x = Distance between P and the line of action of the Resultant force

Now taking moment of the vertical components of the forces and the resultant force about P, and Equating the same

The following forces act @ a point

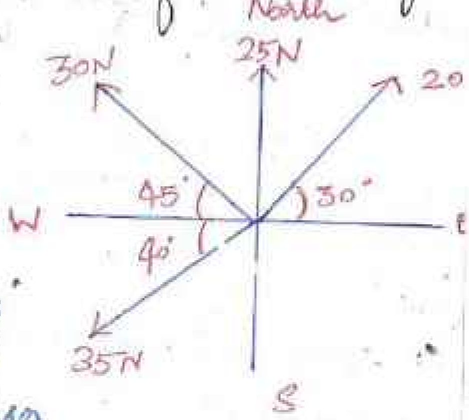
(i) 20N inclined @ 30° towards North of East.

(ii) 25N towards North.

(iii) 30N towards North west,

(iv) 35N inclined @ 40° towards South of west.

Find the magnitude and direction of the resultant force



$\Sigma H = 20\cos 30° - 30\cos 45° - 35\cos 40°$

$= 17.32 - 21.21 - 26.81$

$= -30.7 N$

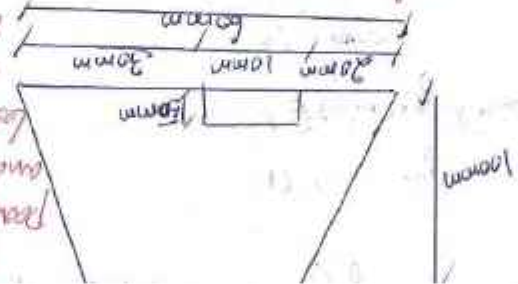
$\Sigma V = 20\sin 30° + 25 + 30\sin 45° - 35\sin 40°$

$= 10 + 25 + 21.21 - 22.49$

$= 33.72 N$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 N$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{33.7}{30.7}\right) = \tan^{-1}(1.097) = 47.7°$



A trapezium has two parallel side 30mm and 60mm, and is positioned with the 60mm side @ the bottom and height of trapezium is 100 mm. A Rectangle hole of width 10mm and height 5cm is cut out from the trapezium with the left Edge of the Rectangle @ distance 20mm from the left most Edge of the trapezium. Find the Center of gravity of the Remaining lamina

Section ① Trapezium

$a_1 = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(30+60)h$

$= \frac{1}{2}(90)(100)$

$= 4500 mm^2$

$x_1 = b/2 = 60/2 = 30mm$

$y_1 = \left(\frac{b+2a}{b+a}\right)h/3$

$= \left(\frac{60+2(30)}{60+30}\right)\frac{100}{3}$

$= \frac{120}{90} \times \frac{100}{3}$

$= 44.44 mm$

Section ② Rectangle

$a_2 = 50 \times 10 = 500 mm^2$

$x_2 = 20 + \frac{10}{2} = 25mm$

$y_2 = \frac{50}{2} = 25mm$

$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$

$= \frac{(4500 \times 30) - (500 \times 25)}{4500 - 500}$

$= \frac{135{,}000 - 12500}{4000}$

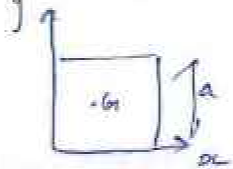
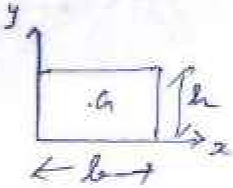
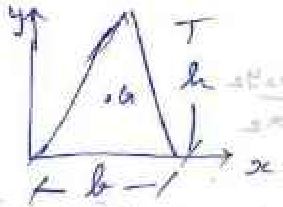
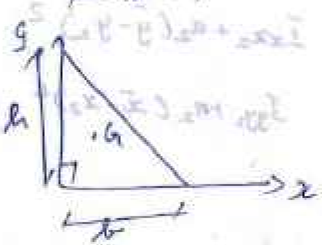
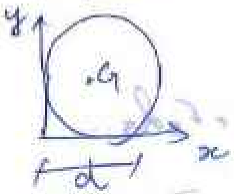
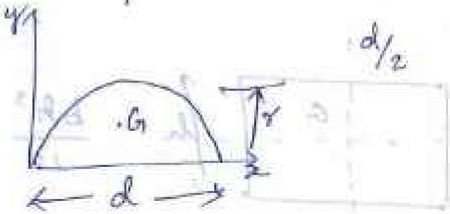
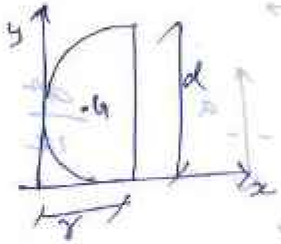
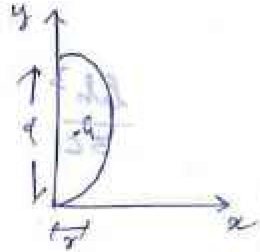
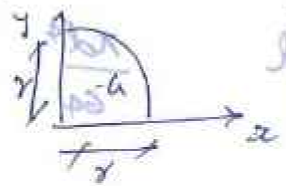
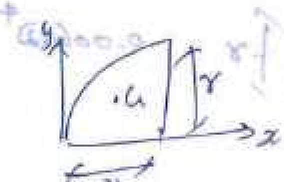
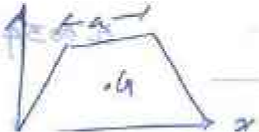
$= 30.625 mm$

$\bar{y} = \frac{a_1y_1 - a_2y_2}{a - a_2} = \frac{(4500 \times 44.49) - (500 \times 25)}{4000}$

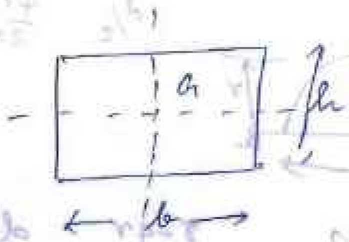
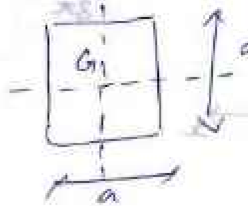
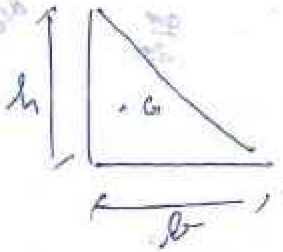
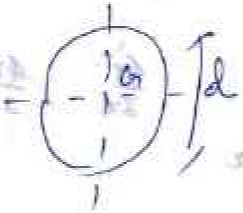
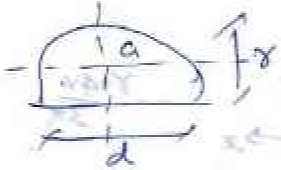
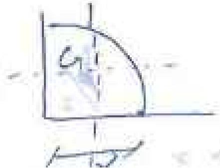
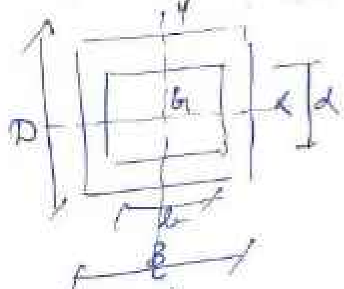
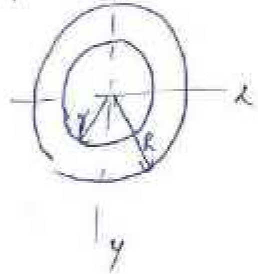
$= 46.87mm$

Centroid of Simple plane figure.

May 29 8.31.

S.NO	Name	Shape	$\bar{x}$	$\bar{y}$	Area
1	Square		$a/2$	$a/2$	a^2
2	Rectangle		$\frac{b}{2}$	$\frac{h}{2}$	$b \times h$
3	Triangle (Isosceles)		$\frac{b}{2}$	$\frac{h}{3}$	$\frac{1}{2}bh$
4	Triangle (Right angled)		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2}bh$
5	Triangle (Right angles)		$\frac{2b}{3}$	$\frac{h}{3}$	$\frac{1}{2}bh$
6	Circle		$d/2$	$d/2$	$\frac{\pi}{4}d^2$
7	Semi circle		$d/2$	$\frac{4r}{3\pi}$	$\frac{1}{2} \times \frac{\pi}{4}d^2$
8	Semi circle		$r - \frac{4r}{3\pi}$	$d/2$	$\frac{1}{2}\frac{\pi}{4}d^2$
9	Semi circle		$\frac{4r}{3\pi}$	$d/2$	$\frac{1}{2}\frac{\pi}{4}d^2$
10	Quadrant		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{1}{4}\frac{\pi}{4}d^2$
11	Quadrant		$r - \frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{1}{4}\frac{\pi}{4}d^2$
12	Trapezium		$b/2$	$\left(\frac{b+2a}{b+a}\right)\frac{h}{3}$	$\frac{1}{2}(a+b)h$

Moment of Inertia For Simple plane Sections

S.No	Name	Shape	I_{xx}	I_{yy}
1	Rectangle		$\frac{bh^3}{12}$	$\frac{hb^3}{12}$
2	square		$\frac{a^4}{12}$	$\frac{a^4}{12}$
3	Triangle		$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
4	Circle		$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{64}$
5	Semi Circle		$0.0068D^4$	$\frac{\pi d^4}{128}$
6	Quarter Circle		$0.0034D^4$	$0.0034D^4$
7	Hollow rectangle		$\frac{1}{12}[BD^3-bd^3]$	$\frac{1}{12}[DB^3-db^3]$
8	Hollow Circle		$\frac{\pi}{4}(R^4-r^4)$	$\frac{\pi}{4}(R^4-r^4)$

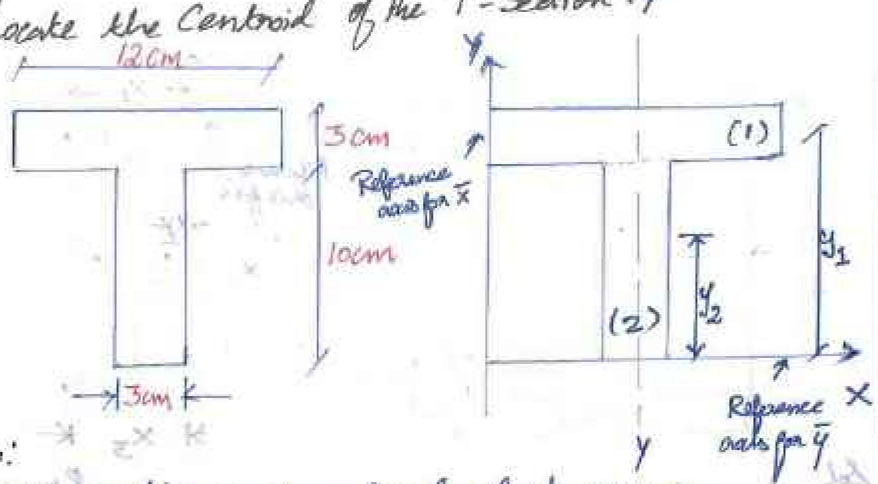
$$\frac{\pi(2r)^4}{128} = \frac{16\pi r^4}{128} = \frac{\pi r^4}{8}$$

$D = 2r$

$0.0068(2r)^4$

$= 0.11R^4$

Q7 Locate the Centroid of the 'T'-Section.



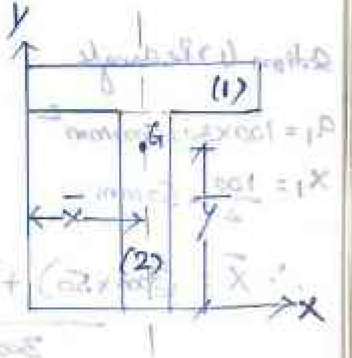
Solution:

The given section is symmetrical about YY axis, therefore its Centroid will lie on this axis. Hence $\bar{x}$ can be found out directly from the geometry on the figure.

The given T section is divided into two Rectangles and the reference axes are drawn on its left and bottom edge of the figure.

Due to symmetry of Y-axis, $\bar{X} = \frac{12}{2} = 6cm$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$



Section (1) Rectangle

Area, $(a_1) = bd = 12 \times 3 = 36cm^2$

$Y_1 = 10 + \frac{3}{2} = 11.5\,cm.$

gap.

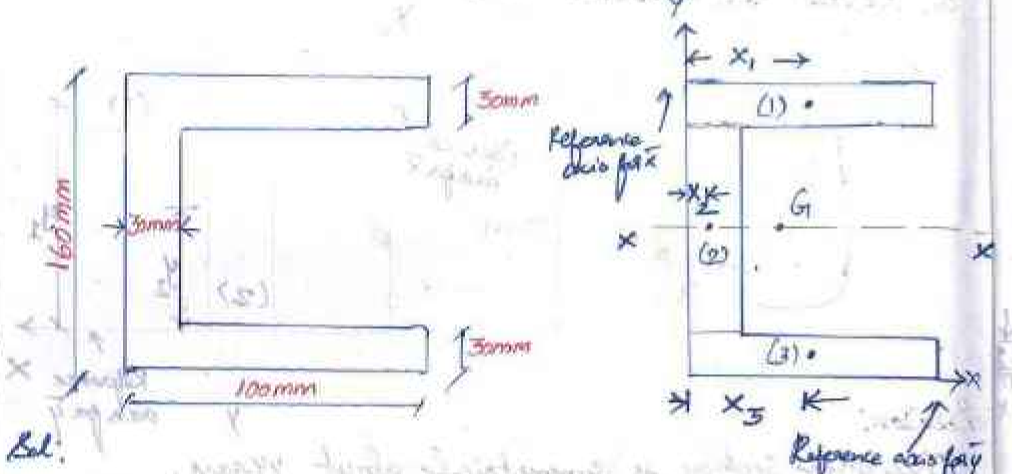
Section (2) Rectangle

Area, $(a_2) = bd = 3 \times 10 = 30cm^2$

$Y_2 = 10/2 = 5cm$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(36 \times 11.5) + (30 \times 5)}{36 + 30} = \frac{414 + 150}{66} = 8.545cm$$

Q) Determine the centre of gravity of the channel section.



Sol:

The given section is symmetrical about x-x axis, therefore its centre of gravity will lie on this axis. Hence $\bar{y}$ can be found directly from the geometry of the figure.

Due to symmetry (X-axis), $\bar{y} = \frac{160}{2} = 80\,mm$

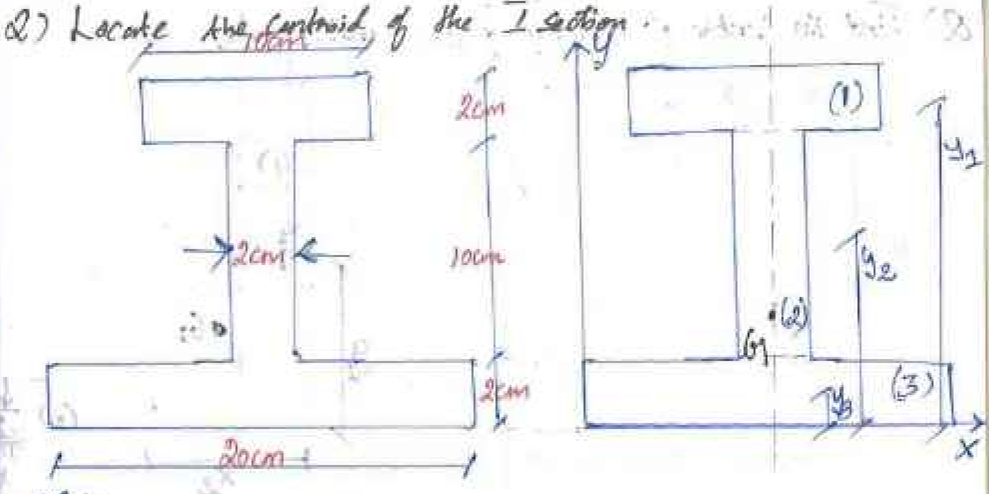
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_2x_3}{a_1 + a_2 + a_3}$$

Section (1) Rectangle	Section (2) Rectangle	Section (3) Rectangle
$a_1 = 100 \times 30 = 3000\,mm^2$ $x_1 = \frac{100}{2} = 50\,mm$	$a_2 = 30 \times 100 = 3000\,mm^2$ $x_2 = \frac{30}{2} = 15\,mm$	$a_3 = 100 \times 30 = 3000\,mm^2$ $x_3 = \frac{100}{2} = 50\,mm$

$$\therefore \bar{x} = \frac{(3000 \times 50) + (3000 \times 15) + (3000 \times 50)}{3000 + 3000 + 3000}$$

$$\bar{x} = \frac{1,50,000 + 45,000 + 1,50,000}{9,000} = 38.33\,mm.$$

Q) Locate the centroid of the I section.



Sol:-

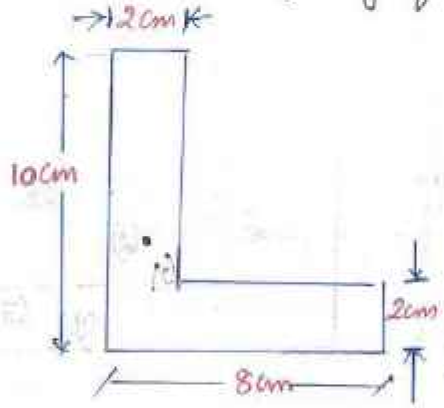
Due to symmetry, $\bar{x} = \frac{20}{2} = 10\,cm$

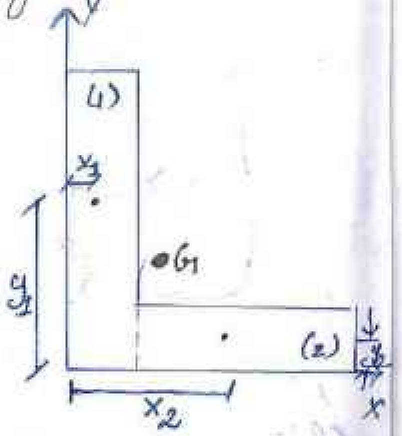
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Section (1) Rectangle	Section (2) Rectangle	Section (3) Rectangle
$a_1 = bd = 10 \times 2 = 20\,cm^2$ $y_1 = 2 + 10 + \frac{2}{2} = 13\,cm$ gap	$a_2 = 2 \times 10 = 20\,cm^2$ $y_2 = 2 + \frac{10}{2} = 7\,cm$ gap	$a_3 = 20 \times 2 = 40\,cm^2$ $y_3 = \frac{2}{2} = 1\,cm$

$$\bar{y} = \frac{(20 \times 13) + (20 \times 7) + (40 \times 1)}{20 + 20 + 40} = \frac{260 + 140 + 40}{80} = 5.5\,cm.$$

Q) Find the centre of gravity of the angle section





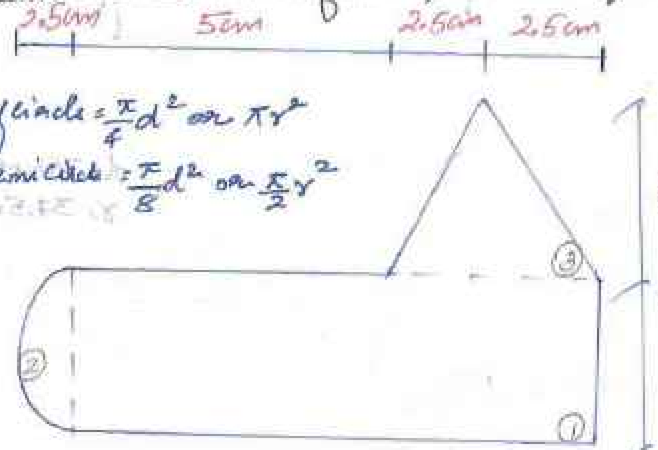
Sol:
The given section is not symmetrical about any axis, therefore we have to find the values of $\bar{x}$ and $\bar{y}$. The given section is divided into two rectangles and the reference axes are drawn on its left and bottom edge.

Section (1) Rectangle	Section (2) Rectangle
Area, $a_1 = 2\times 10 = 20cm^2$	Area, $a_2 = 6\times 2 = 12cm^2$
$x_1 = \frac{2}{2} = 1cm$	$x_2 = 2+\frac{6}{2} = 2+3 = 5cm$
$y_1 = \frac{10}{2} = 5cm$	$y_1 = \frac{2}{2} = 1cm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2}$$
$$= \frac{(20\times 1)+(12\times 5)}{20+12} = \frac{20+60}{32}$$
$$\bar{x} = 2.5cm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2}$$
$$= \frac{(20\times 5)+(12\times 1)}{20+12}$$
$$\bar{y} = \frac{100+12}{32} = \frac{112}{32} = 3.5cm$$

Q) Locate the Centroid of the plane uniform lamina -



$d = 2r$

area of circle $= \frac{\pi}{4}d^2$ or πr^2

semicircle $= \frac{\pi}{8}d^2$ or $\frac{\pi}{2}r^2$

Section (1) Rectangle

$a_1 = 10\times 5 = 50cm^2$

$x_1 = 2.5 + \frac{10}{2} = 7.5cm$

$y_1 = \frac{5}{2} = 2.5cm$

Section (2) Semicircle

$a_2 = \frac{\pi}{2}r^2 = \frac{\pi}{2}\times 2.5^2 = 9.82cm^2$

$x_2 = r - \frac{4r}{3\pi}$

$= 2.5 - \frac{(4\times 2.5)}{3\pi}$

$= 2.5 - 1.06 = 1.44cm$

$y_2 = \frac{5}{2} = 2.5$

Section (3) Isosceles triangle

$a_3 = \frac{1}{2}bh$

$= \frac{1}{2}\times 5\times 5$

$= 12.5cm^2$

$x_3 = 2.5+5+\frac{5}{2}$

$= 10cm$

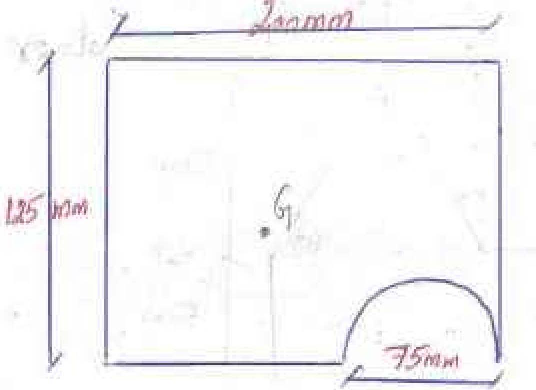
$y_3 = 5+\frac{5}{3}$

$= 6.67cm$

$$\bar{x} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3} = \frac{(50\times 7.5)+(9.82\times 1.44)+(12.5\times 10)}{50+9.82+12.5}$$
$$= \frac{375+14.141+125}{72.32} = \frac{514.14}{72.32}$$
$$\boxed{\bar{x} = 7.11cm}$$

$$\bar{y} = \frac{a_1y_1+a_2y_2+a_3y_3}{a_1+a_2+a_3} = \frac{(50\times 2.5)+(9.82\times 2.5)+(12.5\times 6.67)}{50+9.82+12.5}$$
$$= \frac{125+24.55+83.375}{72.32} = \frac{232.93}{72.32}$$

Q) Locate the centroid of the lamina shown in fig.



$d = 75mm$
$r = 37.5mm$

sol:- The given section is not symmetrical about any axis, therefore we have to find the value of $\bar{x}$ and $\bar{y}$. The given section is divided into Section (1) as a rectangle and section (2) as a semicircle and the reference axes are drawn on its left and bottom edge.

Section (1) Rectangle	Section (2) Semicircle
$a_1 = 200 \times 125 = 25{,}000 mm^2$	$a_2 = \frac{\pi R^2}{2} = \frac{\pi \times 37.5^2}{2} = 2208.93 mm^2$
$x_1 = \frac{200}{2} = 100mm$	$x_2 = 200 - \frac{75}{2} = 162.5mm$ (gap)
$y_1 = \frac{125}{2} = 62.5mm$	$y_2 = \frac{4R}{3\pi} = \frac{4 \times 37.5}{3 \times \pi} = 15.92mm$

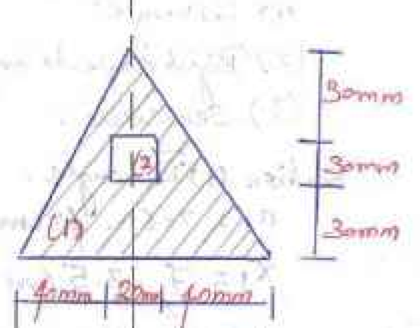
$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{(25000 \times 100) - (2208.93 \times 162.5)}{25000 - 2208.93} = \frac{2141048.9}{22791.07}$$

$= 93.94mm$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(25000 \times 62.5) - (2208.93 \times 15.92)}{25000 - 2208.93} = \frac{1527{,}333.8}{22791.07}$$

$= 67.01mm$

Q) Find the centroid of the section shown in fig.



Due to symmetry $\bar{x} = \frac{100}{2} = 50mm$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

Section(1) Triangle

$a_1 = \frac{1}{2} bh = \frac{1}{2} \times 100 \times 90$

$= 4500 mm^2$

$y_1 = \frac{h}{3} = \frac{90}{3} = 30mm$

Section(2) Rectangle

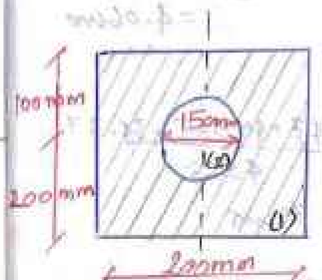
$a_2 = 20 \times 30 = 600 mm^2$

$y_2 = \underbrace{30}_{gap} + \frac{30}{2} = 45mm$

$$\bar{y} = \frac{(4500 \times 30) - (600 \times 45)}{4500 - 600} = \frac{135000 - 27000}{3900} = \frac{108000}{3900}$$

$\bar{y} = 27.69mm$

Q) Find the centroid of a hollow section shown in fig.



Due to symmetry, $\bar{x} = \frac{200}{2} = 100mm$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

Section(1) Rectangle

$a_1 = 300 \times 200 = 60000 mm^2$

$y_1 = \frac{300}{2} = 150mm$

Section(2) Circle

$a_2 = \pi R^2 = \pi \times 75^2$

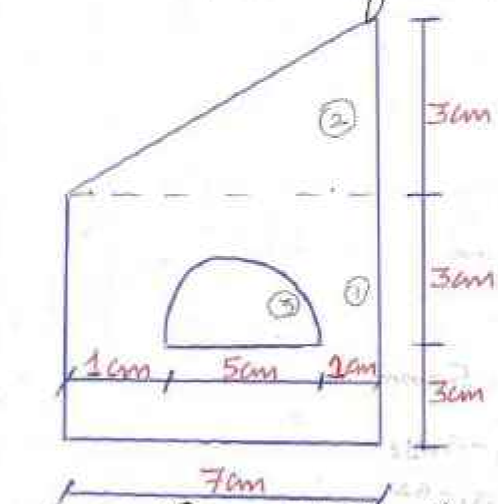
$= 17670 mm^2$

$y_2 = 200$ ← Value are directly given in fig

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(60000 \times 150) - (17670 \times 200)}{60000 - 17670}$$

$\bar{y} = 120 mm$

Q) Find the Centroid of the area Shown in fig.



(1) Rectangle (+)
(2) Right angle triangle (+)
(3) Semicircle (−)

Section (1) Rectangle.

$a_1 = 7\times6 = 42\,cm^2$

$x_1 = \frac{7}{2} = 3.5\,cm$

$y_1 = 6/2 = 3cm$

$d = 2r$

Section (2) Right angled Triangle

$a_2 = \frac{1}{2} bh = \frac{1}{2}\times7\times3 = 10.5cm^2$

$x_2 = \frac{2}{3}b = \frac{2}{3}\times7 = 4.67cm$

$y_2 = 6+\frac{h}{3} = 6+\frac{3}{3} = 7cm$

gap.

Section (3) Semicircle

$a_3 = \frac{\pi R^2}{2} = \frac{1}{2}\times\frac{\pi}{4}d^2 = \frac{1}{2}\times\frac{\pi}{4}\times4r^2$

$= \frac{\pi}{2}(2.5)^2$

$a_3 = 9.82\,cm^2$

$x_3 = 1+\frac{d}{2} = 1+\frac{5}{2} = 3.5cm$

gap

$y_3 = 3+\frac{4r}{3\pi} = 3+\left(\frac{4\times2.5}{3\pi}\right)$

$= 4.06cm$

$\therefore \bar{x} = \frac{a_1x_1+a_2x_2-a_3x_3}{a_1+a_2-a_3}$

$= \frac{(42\times3.5)+(10.5\times4.67)-(9.82\times3.5)}{42+10.5-9.82}$

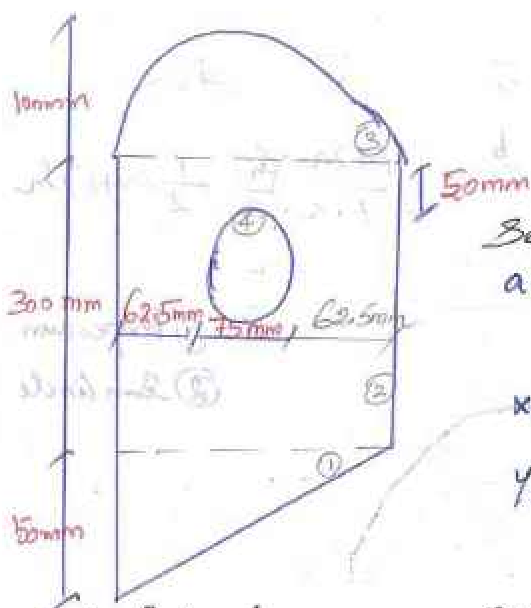
$= \frac{147+49.035-34.37}{42.68}$

$= 3.79cm$

$\therefore \bar{y} = \frac{a_1y_1+a_2y_2-a_3y_3}{a_1+a_2-a_3} = \frac{(42\times3)+(10.5\times7)-(9.82\times4.06)}{42+10.5-9.82}$

$= \frac{126+73.5-39.86}{42.68} = 3.74cm.$

Q) Determine the location of the centroid of the plane area Shown in fig



(1) Triangle (+)
(2) Rectangle (+)
(3) Semicircle (+)
(4) circle − (−)

Section (1) Triangle

$a_1 = \frac{1}{2}bh = \frac{1}{2}\times200\times50$

$= 5000\,mm^2$

$x_1 = \frac{b}{3} = \frac{200}{3} = 66.67mm$

$y_1 = \frac{2}{3}h = \frac{2}{3}\times50 = 33.33mm$

Section (2) Rectangle

$a_2 = 200\times300 = 60000\,mm^2$

$x_2 = \frac{200}{2} = 100mm$

$y_2 = 50+\frac{300}{2} = 200mm$

gap

Section (3) Semicircle

$a_3 = \frac{1}{2}\times\frac{\pi}{4}d^2 = \frac{\pi r^2}{2} = \frac{\pi}{2}\times100^2$

$= 15707.96 = 15708\,mm^2$

$x_3 = \frac{d}{2} = \frac{200}{2} = 100mm$

$y_3 = 50+300+\frac{4r}{3\pi}$ ← 100.

gap

$= 392.44mm$

Section (4) Circle

$a_4 = \frac{\pi}{4}d^2 = \pi r^2 = \pi\times37.5^2$

$= 4417.86mm^2$

$x_4 = 62.5+\frac{d}{2} = 62.5+\frac{75}{2} = 100mm$

$y_4 = 50+175+\frac{d}{2} = 50+175+\frac{75}{2}$

$= 262.5mm$

or

$y_4 = 50+(300-50-75/2)$

$= 262.5mm$

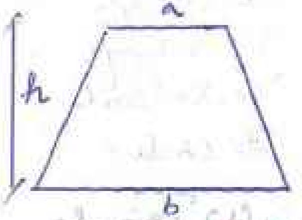
$\bar{X} = \frac{a_1x_1+a_2x_2+a_3x_3-a_4x_4}{a_1+a_2+a_3-a_4}$

$= 97.82mm$

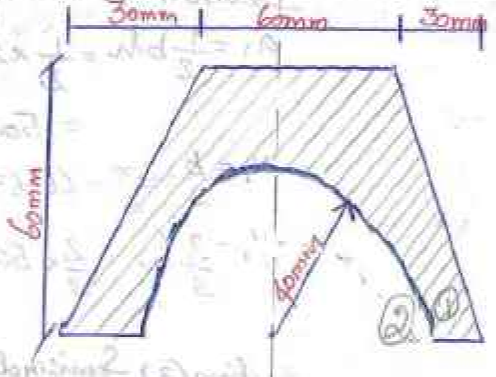
$\bar{Y} = \frac{a_1y_1+a_2y_2+a_3y_3-a_4y_4}{a_1+a_2+a_3-a_4}$

$= 225.07mm.$

Q) Find the Centroid of the Shaded area Shown in fig



$\bar{x}$	$\bar{y}$	Area
$\frac{b}{2}$	$\left(\frac{b+2a}{b+a}\right)\frac{h}{3}$	$\frac{1}{2}(a+b)h$



(1) Trapezium (+)
(2) Semi circle (−)

Sol:-

Due to symmetry, $\bar{x} = \frac{120}{2} = 60mm$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

Section (1) Trapezium

$$a_1 = \frac{1}{2}(a+b)h = \frac{1}{2}(60+120)60 = 5400mm^2$$

$$y_1 = \left(\frac{b+2a}{b+a}\right)\frac{h}{3} = \left(\frac{120+2(60)}{120+60}\right)\frac{60}{3} = 26.67mm$$

Section (2) Semicircle

$$a_2 = \frac{\pi r^2}{2} = \frac{\pi \times 40^2}{2} = 2513.27mm^2$$

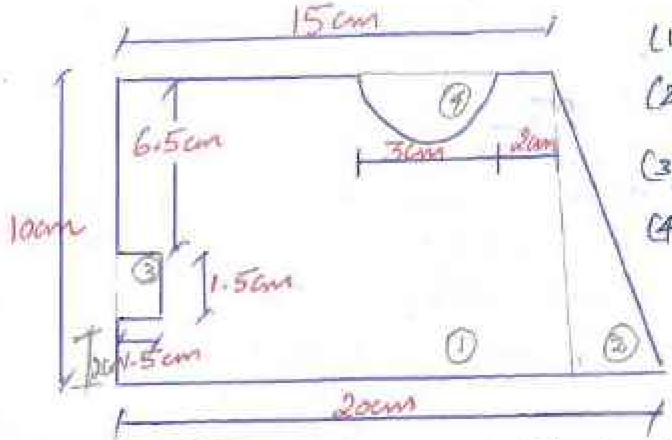
$$y_2 = \frac{4r}{3\pi} = \frac{4\times 40}{3\pi} = 16.97 \approx 17mm$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(5400\times 26.67) - (2513.27\times 17)}{5400 - 2513.27}$$

$$= \frac{1,44,018 - 42,725.59}{2886.73}$$

$$\bar{y} = 35.08 mm.$$

Q) Locate the centroid of area Shown in fig.



(1) Rectangle (+)
(2) Right angled Triangle (+)
(3) Square (−)
(4) Semicircle (−)

Section (1) Rectangle

$a_1 = 10\times 15 = 150cm^2$

$x_1 = \frac{15}{2} = 7.5cm$

$y_1 = \frac{10}{2} = 5cm$

Section (2) Right Angled Triangle

$a_2 = \frac{1}{2}bh = \frac{1}{2}\times 5\times 10 = 25cm^2$

$x_2 = 15 + \frac{b}{3} = 15 + \frac{5}{3} = 16.67cm$

$y_2 = \frac{h}{3} = \frac{10}{3} = 3.33cm$

Section (3) Square

$a_3 = 1.5\times 1.5 = 2.25cm^2$

$x_3 = \frac{1.5}{2} = 0.75cm$

$y_3 = \underset{gap}{2} + \frac{1.5}{2} = 2.75cm$

Section (4) Semicircle

$a_4 = \frac{\pi r^2}{2} = \frac{\pi}{2}\times 1.5^2 = 3.53cm^2$

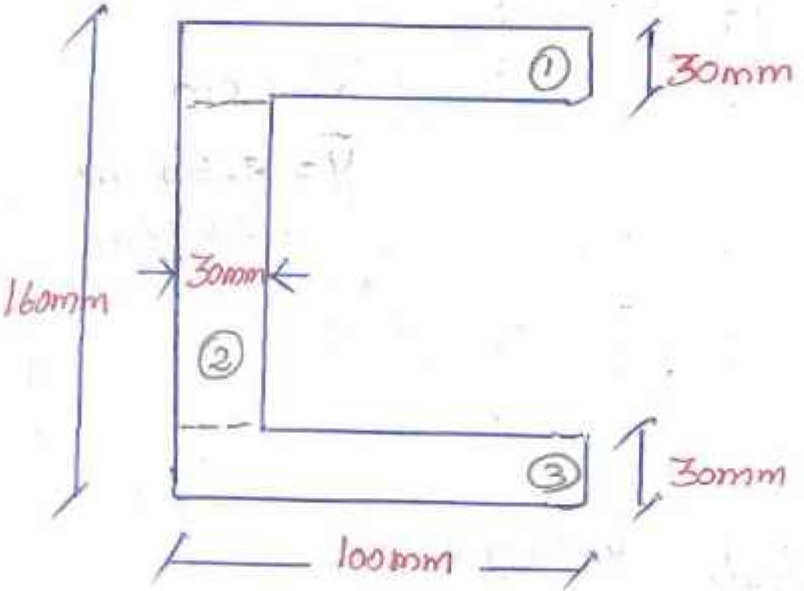
$x_4 = \underset{8+2}{10} + \frac{d}{2} = 10 + \frac{3}{2} = 11.5cm^2$

$y_4 = 10 - \frac{4r}{3\pi} = 10 - \frac{4\times 1.5}{3\pi} = 9.36cm.$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3 - a_4x_4}{a_1 + a_2 - a_3 - a_4} = \frac{1409.467}{169.22} = 8.861cm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3 - a_4y_4}{a_1 + a_2 - a_3 - a_4} = \frac{794.02}{169.22} = 4.69cm.$$

Q) Find the moment of Inertia of the channel section shown in fig about its centroidal axes.



Sol:

Due to symmetry, $\bar{Y} = \frac{160}{2} = 80\,mm$

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

Section(1) Rectangle	Section (2) Rectangle	Section (3) Rectangle.
100mm × 30mm	100 mm × 30mm	30mm. × 100mm
Area, $a_1 = 100 \times 30$ $= 3000\,mm^2$	$a_2 = 30 \times 100$ $= 3000\,mm^2$	$a_3 = 100 \times 30$ $= 3000\,mm^2$
$x_1 = \frac{100}{2} = 50\,mm$	$x_2 = \frac{30}{2} = 15\,mm$	$x_3 = \frac{100}{2} = 50\,mm$

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{1{,}50{,}000 + 45{,}000 + 1{,}50{,}000}{9{,}000} = 38.33\,mm$$

To find the moment of inertia about the centroidal axes (xx and yy).

Section	$(I_{self})_{xx}$	$(I_{self})_{yy}$	Area (a)	$a(x-\bar{x})^2$	$a(y-\bar{y})^2$
1	$\frac{bd^3}{12} = \frac{100\times30^3}{12}$ $= 225,000$	$\frac{db^3}{12} = \frac{30\times100^3}{12}$ $= 2,500,000$	100×30 $= 3000$	$a_1(x_1-\bar{x})^2$ $= 3000(50-38.33)^2$ $= 408.57\times10^3$	$a_1(y_1-\bar{y})^2$ $= 3000(145-80)^2$ $= 12.675\times10^6$
2	$\frac{bd^3}{12} = \frac{30\times100^3}{12}$ $= 2,500,000$	$\frac{db^3}{12} = \frac{100\times30^3}{12}$ $= 225,000$	30×100 $= 3000$	$a_2(x_2-\bar{x})^2$ $= 3000(15-38.33)^2$ $= 1.63\times10^6$	$a_2(y_2-\bar{y})^2$ $= 3000(80-80)^2$ $= 0$
3	$\frac{bd^3}{12} = \frac{100\times30^3}{12}$ $= 2,25,000$	$\frac{db^3}{12} = \frac{30\times100^3}{12}$ $= 2,500000$	100×30 $= 3000$	$a_3(x_3-\bar{x})^2$ $= 3000(50-38.33)^2$ $= 408.57\times10^3$	$a_3(y_3-\bar{y})^2$ $= 3000(15-80)^2$ $= 12.675\times10^6$
	$\Sigma(I_{self})_{xx}$ $= 2,950000$	$\Sigma(I_{self})_{yy}$ $= 5,225,000$		$\Sigma a(x-\bar{x})^2$ $= 2.447\times10^6$ $= 2.45\times10^6$	$\Sigma a(y-\bar{y})^2$ $= 25.35\times10^6$

Moment of Inertia about xx axis, $I_{xx} = \Sigma(I_{self})_{xx} + \Sigma a(y-\bar{y})^2$

$I_x = A(\bar{y})^2$

$$= (2,950,000) + (25.35\times10^6)$$

$$\boxed{I_{xx} = 28.3\times10^6\ mm^4}$$

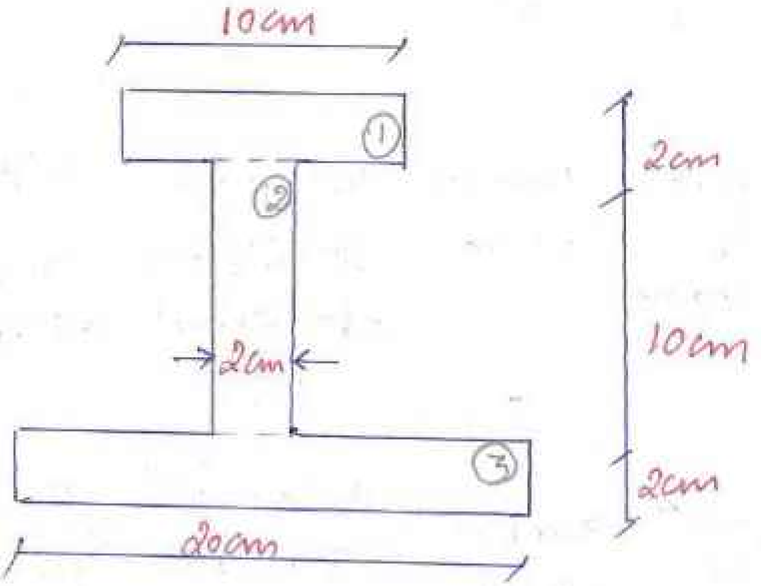
Moment of Inertia about yy axis, $I_{yy} = \Sigma(I_{self})_{yy} + \Sigma a(x-\bar{x})^2$

$I_y = A(\bar{x})^2$

$$= (5,225,000) + (2.45\times10^6)$$

$$\boxed{I_{yy} = 7.675\times10^6\ mm^4}$$

Find the moment of Inertia of the I section shown in fig about Centroidal axes.



Sol: The given section is symmetrical about yy axis, therefore its centroid will lie on this axis. Hence $\bar{x}$ can be found out directly from the geometry of the figure.

Due to symmetry, $\bar{x} = \frac{20}{2} = 10cm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = 5.5cm$$

Section ① Rectangle	Section ② Rectangle	Section ③ Rectangle
Area, $a_1 = bd = 10 \times 2 = 20cm^2$ $y_1 = 2 + 10 + \frac{2}{2}$ $= 13cm$	Area, $a_2 = bd = 10 \times 2 = 20cm^2$ $y_2 = 2 + \frac{10}{2}$ $= 7cm$	Area, $a_3 = bd = 20 \times 2$ $= 40cm^2$ $y_3 = \frac{2}{2}$ $= 1cm$

$I_{xx} = A \times (\bar{y})^2$

$I_{yy} = A \times (\bar{x})^2$

To find the moment of Inertia about the Centroidal axes (xx and yy)

Section	$(I_{self})_{xx}$ cm^4	$(I_{self})_{yy}$ cm^4	Area (a) cm^2	$a(x-\bar{x})^2$ cm^4	$a(y-\bar{y})^2$ cm^4
1	$\frac{bd^3}{12} = \frac{10\times2^3}{12} = 6.67$	$\frac{db^3}{12} = \frac{2\times10^3}{12} = 166.67$	$10\times2 = 20$	$a_1(x_1-\bar{x})^2 = 20(10-10)^2 = 0$	$a_1(y_1-\bar{y})^2 = 20(13-5.5)^2 = 1125.$
2	$\frac{bd^3}{12} = \frac{2\times10^3}{12} = 166.67$	$\frac{db^3}{12} = \frac{10\times2^3}{12} = 6.67.$	$10\times2 = 20$	$a_2(x_2-\bar{x})^2 = 20(10-10)^2 = 0$	$a_2(y_2-\bar{y})^2 = 20(7-5.5)^2 = 45.$
3	$\frac{bd^3}{12} = \frac{20\times2^3}{12} = 13.33$	$\frac{db^3}{12} = \frac{2\times20^3}{12} = 1333.33$	$2\times20 = 40$	$a_3(x_3-\bar{x})^2 = 40(10-10)^2 = 0$	$a_3(y_3-\bar{y})^2 = 40(1-5.5)^2 = 810.$
	$\Sigma(I_{self})_{xx} = 186.67$	$\Sigma(I_{self})_{yy} = 1506.67$		$\Sigma a(x-\bar{x})^2 = 0$	$\Sigma a(y-\bar{y})^2 = 1980.$

Moment of Inertia about xx axis, $I_{xx} = \Sigma(I_{self})_{xx} + \Sigma a(y-\bar{y})^2$

$= 186.67 + 1980$

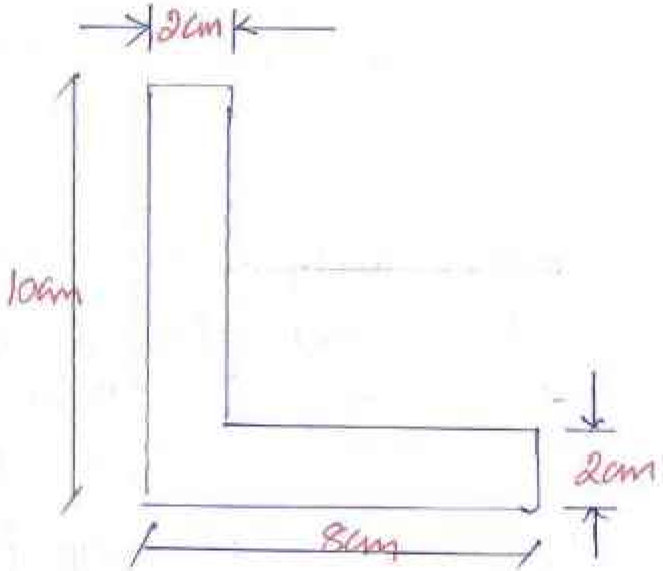
$I_{xx} = 2166.67 \, cm^4$

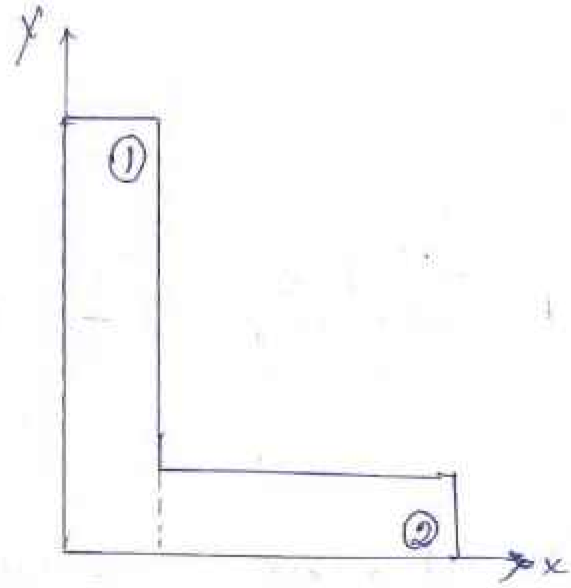
Moment of Inertia about yy axis, $I_{yy} = \Sigma(I_{self})_{yy} + \Sigma a(x-\bar{x})^2$

$= 1506.67 + 0$

$I_{yy} = 1506.67 \, cm^4$

Find the moment of Inertia of the angle Section shown in fig about its Centroidal axes.





Sol:

The given Section is not Symmetrical about any axis, therefore we have to find the values of $\bar{x}$ and $\bar{y}$. The given Section is divided into two rectangles and the reference axes are drawn on its left and bottom edge.

Section ① Rectangle	Section ② Rectangle
Area, $a_1 = 2 \times 10 = 20\ cm^2$ $x_1 = \frac{2}{2} = 1cm$ $y_1 = \frac{10}{2} = 5cm$	Area, $a_2 = 6 \times 2 = 12cm^2$ $x_2 = 2 + \frac{6}{2} = 5cm$ $y_2 = \frac{2}{2} = 1cm.$

$$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(20 \times 1) + (12 \times 5)}{20 + 12}$$

$$= \frac{20 + 60}{32}$$

$$\bar{X} = 2.5cm$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{(20 \times 5) + (12 \times 1)}{20 + 12}$$

$$= \frac{100 + 12}{32} = \frac{112}{32}$$

$$\bar{Y} = 3.5cm.$$

To find the moment of Inertia about the centroidal axes (xx and yy).

Section	$(I_{self})_{xx}$ cm^4	$(I_{self})_{yy}$ cm^4	Area (a) cm^2	$a(x-\bar{x})^2$ cm^4	$a(y-\bar{y})^2$ cm^4
1	$\frac{bd^3}{12} = \frac{2\times10^3}{12} = 166.67$	$\frac{db^3}{12} = \frac{10\times2^3}{12} = 6.67$	$2\times10 = 20$	$a_1(x_1-\bar{x})^2 = 20(1-2.5)^2 = 45$	$a_1(y_1-\bar{y})^2 = 20(5-3.5)^2 = 45$
2	$\frac{bd^3}{12} = \frac{6\times2^3}{12} = 4$	$\frac{db^3}{12} = \frac{2\times6^3}{12} = 36$	$6\times2 = 12$	$a_2(x_2-\bar{x})^2 = 12(5-2.5)^2 = 75$	$a_2(y_2-\bar{y})^2 = 12(1-3.5)^2 = 75$
	$\Sigma(I_{self})_{xx} = 170.67$	$\Sigma(I_{self})_{yy} = 42.67$		$\Sigma a(x-\bar{x})^2 = 120$	$\Sigma a(y-\bar{y})^2 = 120$

Moment of Inertia about xx axis, $I_{xx} = \Sigma(I_{self})_{xx} + \Sigma a(y-\bar{y})^2$

$$= 170.67 + 120$$

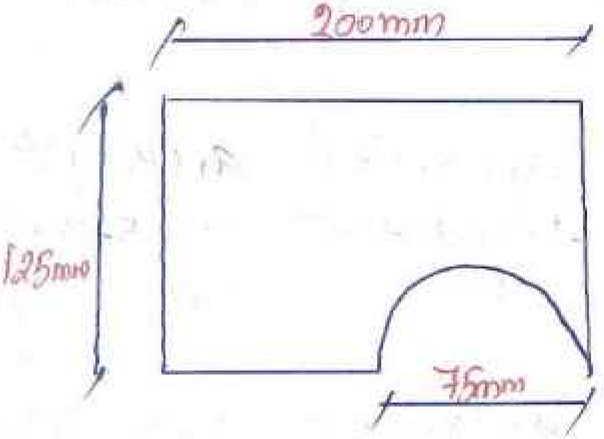
$$I_{xx} = 290.67\ cm^4$$

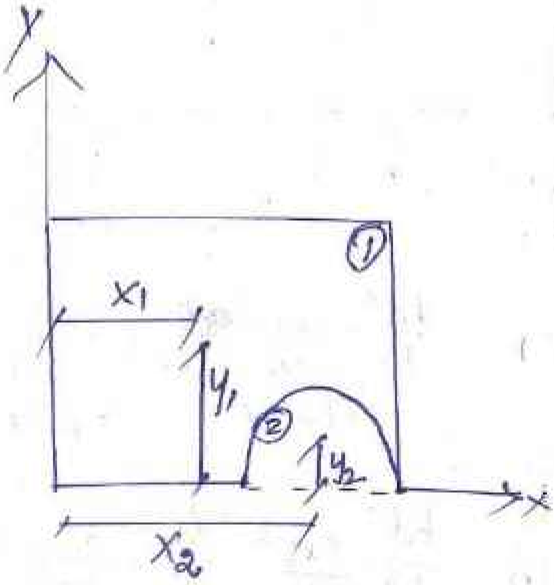
Moment of Inertia about yy axis, $I_{yy} = \Sigma(I_{self})_{yy} + \Sigma a(x-\bar{x})^2$

$$= 42.67 + 120$$

$$I_{yy} = 162.67\ cm^4$$

Find the moment of inertia of the lamina shown in Fig about its centroidal axes.





Sol:

The given section is not symmetrical about any axis, therefore we have to find the value of $\bar{x}$ and $\bar{y}$. The given section is divided into section (1) as a rectangle and section (2) as a semicircle and the reference axes are drawn on its left and bottom edge.

Section ① Rectangle	Section ② Semicircle
$a_1 = 200 \times 125 = 25{,}000\,mm^2$ $x_1 = \frac{200}{2} = 100mm$ $y_1 = \frac{125}{2} = 62.5mm$	$a_2 = \frac{\pi r^2}{2} = \frac{\pi}{2} \times 37.5^2 = 2208.93mm^2$ $x_2 = 200 - \frac{75}{2} = 162.5mm$ gap $y_2 = \frac{4r}{3\pi} = \frac{4 \times 37.5}{3\pi} = 15.92\,mm$

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{(2500 \times 100) - (2208.93 \times 162.5)}{25000 - 2208.93} = \frac{2141048.9}{22791.07}$$

$$= 93.94mm.$$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(25000 \times 62.5) - (2208.93 \times 15.92)}{2500 - 2208.93} = \frac{1527{,}333.8}{22791.07}$$

$$= 67.01\,mm$$

To find the moment of Inertia about the Centroidal axes (xx & yy)

Section	$(I_{self})_{xx}$ mm^4	$(I_{self})_{yy}$ mm^4	Area (a) mm^2	$a(x-\bar{x})^2$ mm^4	$a(y-\bar{y})^2$ mm^4
1	$\frac{bd^3}{12} = \frac{200 \times 125^3}{12}$ $= 32.515 \times 10^6$	$\frac{db^3}{12} = \frac{200^3 \times 125}{12}$ $= 83.33 \times 10^6$	200×125 $= 25,000 mm^2$	$a_1(x_1-\bar{x})^2$ $= 25,000(100-93.94)^2$ $= 918.09 \times 10^3$	$a_1(y_1-\bar{y})^2$ $= 25,000(62.5-67.01)^2$ $= 508 \times 10^3$
2. (−)	$0.0068 D^4$ $= 0.0068(75)^4$ $= 215.15 \times 10^3$	$\frac{\pi d^4}{128} = \frac{\pi(75)^4}{128}$ $= 776.57 \times 10^3$	$\frac{1}{2}\frac{\pi}{4}d^2 = \frac{1 \times \pi \times 75^2}{24}$ $= 2208.93$	$a_2(x_2-\bar{x})^2$ $= 2208.93(162.5-93.94)^2$ $= 10.38 \times 10^6$	$a_2(y_2-\bar{y})^2$ $2208.93(15.92-67.01)^2$ 5.76×10^6
	$\Sigma(I_{self})_{xx}$ $= 32.33 \times 10^6$	$\Sigma(I_{self})_{yy}$ $= 82.55 \times 10^6$		$\Sigma a(x-\bar{x})^2$ $= -9.46 \times 10^6$	$\Sigma a(y-\bar{y})^2$ $= -5.256 \times 10^6$.

$$I_x = A \times (\bar{y})^2$$

Moment of Inertia about xx axis, $I_{xx} = \Sigma(I_{self})_{xx} + \Sigma a(y-\bar{y})^2$

$$= (32.33 \times 10^6) - (5.256 \times 10^6)$$

$$\boxed{I_{xx} = 27.074 \times 10^6 mm^4}$$

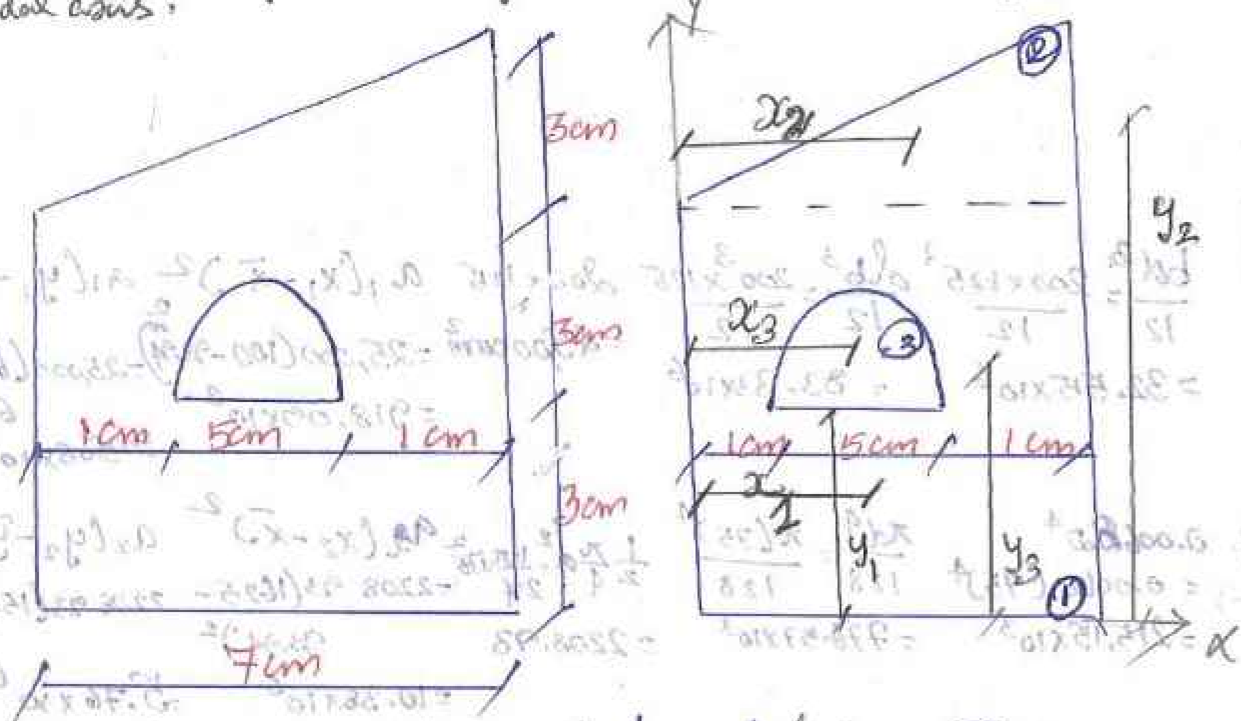
Moment of Inertia about yy axis ; $I_{yy} = \Sigma(I_{self})_{yy} + \Sigma a(x-\bar{x})^2$

$$= (82.55 \times 10^6) - (9.46 \times 10^6)$$

$$\boxed{= 73.09 \times 10^6 mm^4}$$

$$I_y = A \times (\bar{x})^2$$

Find the moment of Inertia of the section shown in fig about its Centroidal axis.



(1) Rectangle (+) 2) Right angle triangle (+) ③ Semicircle (−)

Section (1) Rectangle	Section (2) Right angle triangle	Section ③ Semicircle.
$a_1 = 7 \times 6 = 42 cm^2$ $x_1 = \frac{7}{2} = 3.5 cm$ $y_1 = \frac{6}{2} = 3 cm$	$a_2 = \frac{1}{2} bh = \frac{1}{2} \times 7 \times 3 = 10.50 m^2$ $x_2 = \frac{2}{3} b = \frac{2}{3} \times 7 = 4.67 cm$ $y_2 = \underset{gap}{6} + \frac{h}{3} = 6 + \frac{3}{3} = 7 cm$	$a_3 = \frac{\pi R^2}{2} = \frac{1}{2} \times \frac{\pi}{4} d^2 = \frac{1}{2} \times \frac{\pi}{4} (5)^2 = 9.82 cm^2$ $x_3 = \underset{gap}{1} + \frac{d}{2} = 1 + \frac{5}{2} = 3.50 cm$ $y_3 = 3 + \frac{4r}{3\pi} = 3 + \left(\frac{4 \times 2.5}{3\pi}\right) = 4.06 cm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3}{a_1 + a_2 - a_3} = \frac{(42 \times 3.5) + (10.5 \times 4.67) - (9.82 \times 3.5)}{42 + 10.5 - 9.82}$$

$$= \frac{147 + 49.035 - 34.37}{42.68} = 3.79 cm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3} = \frac{(42 \times 3) + (10.5 \times 7) - (9.82 \times 4.06)}{42 + 10.5 - 9.82} = 3.74 cm.$$

To find the moment of inertia about the centroidal axes (xx and yy).

Section	$(I_{self})_{xx}$ cm^4	$(I_{self})_{yy}$ cm^4	Area (a) cm^2	$a(x-\bar{x})^2$ cm^4	$a(y-\bar{y})^2$ cm^4
1	$\frac{bd^3}{12} = \frac{7\times6^3}{12} = 126$	$\frac{db^3}{12} = \frac{6\times7^3}{12} = 171.5$	$7\times6 = 42$	$a_1(x_1-\bar{x})^2 = 42(3.5-3.79)^2 = 3.53$	$a_1(y_1-\bar{y})^2 = 42(3-3.74)^2 = 22.10.$
2	$\frac{bh^3}{36} = \frac{7\times3^3}{36} = 5.25$	$\frac{hb^3}{36} = \frac{3\times7^3}{36} = 28.58$	$\frac{1}{2}bh = \frac{1}{2}\times7\times3 = 10.5$	$a_2(x_2-\bar{x})^2 = 10.5(4.67-3.79)^2 = 8.13$	$a_2(y_2-\bar{y})^2 = 10.5(7-3.74)^2 = 111.59$
3 (–)	$0.0068d^4 = 0.0068\times5^4 = 4.25$	$\frac{\pi d^4}{128} = \frac{\pi\times5^4}{128} = 15.34$	$\frac{\pi r^2}{2} = \frac{\pi\times2.5^2}{2} = 9.82$	$a_3(x_3-\bar{x})^2 = 9.82(3.5-3.79)^2 = 0.83$	$a_3(y_3-\bar{y})^2 = 9.82(4.06-3.74)^2 = 1.00$
	$\Sigma(I_{self})_{xx} = 127$	$\Sigma(I_{self})_{yy} = 184.74$		$\Sigma a(x-\bar{x})^2 = 10.83$	$\Sigma a(y-\bar{y})^2 = 132.69$

Moment of Inertia about xx axis, $I_{xx} =$

$$\Sigma(I_{self})_{xx} + \Sigma a(y-\bar{y})^2$$

$$= 127 + 132.68$$

$$\boxed{I_{xx} = 259.68\ cm^4}$$

Moment of Inertia about yy axis, $I_{yy} =$

$$\Sigma(I_{self})_{yy} + \Sigma a(x-\bar{x})^2$$

$$= 184.74 + 10.83$$

$$\boxed{I_{yy} = 195.57\ cm^4}.$$

Determine the change in length, breadth and thickness of a steel bar which is 5 m long, 20mm wide and 15 mm thick subjected to an axial pull of 100kN in the direction of its length. Take E = 200 Gpa and poisson's ratio = 0.3.

Given:

Mega = 10^6

Pa = N/m^2

length, L = 5m = 5000 mm

Breadth, b = 20mm

1 kN = 1000 N

Thickness, t = 15mm

Axial pull, P = 100kN = 100×10^3 N

Young's modulus, E = 200 Gpa = 200×10^9 pa

$= 200 \times 10^9 \ N/m^2$

1m = 1000mm

$= 200 \times 10^3 \ N/mm^2$

Poisson's Ratio, (μ) or $(1/m)$ = 0.3

To Find:
- change in length, δl
- change in Breadth, δb
- change in thickness, δt

Sol:

Stress (σ) = $\frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{100 \times 10^3}{b \times t} = \frac{100 \times 10^3}{20 \times 15} = 333.3 \ \frac{N}{mm^2}$

Young's Modulus, E = $\frac{\text{Stress}(\sigma)}{\text{Strain}(e)}$

$$E = \frac{333.3}{e}$$

$$e = \frac{333.3}{200 \times 10^3}$$

$$\boxed{e = 1.667 \times 10^{-3}}$$

$$\text{Strain } (e) = \frac{\text{Change in Length}}{\text{Original length}} = \frac{\delta l}{l}$$

$$e = \frac{\delta l}{l}$$

$$1.667 \times 10^{-3} = \frac{\delta l}{5000}$$

$$\delta l = 1.667 \times 10^{-3} \times 5000$$

$$\boxed{\delta l = 8.335 mm.}$$

$$\text{Poisson's Ratio } (\mu \text{ or } 1/m) = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\delta b / b}{\delta l / l.}$$

$$\mu = \frac{\delta b}{b} \times \frac{l}{\delta l}$$

$$0.3 = \frac{\delta b}{20} \times \frac{5000}{8.335}$$

$$\boxed{\delta b = 10.00 \times 10^{-3} mm}$$

$$\text{lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d} \text{ or } \frac{\delta t}{t}$$

$$\underset{0.3}{\mu} = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{e_{lateral}}{1.667 \times 10^{-3}}$$

$$e_{lateral} = 0.3 \times 1.667 \times 10^{-3}$$

$$\boxed{e_{lateral} = 5.001 \times 10^{-4} \text{ or } 0.500 \times 10^{-3}}$$

$$e_{lateral} = \frac{\delta t}{t} \qquad 0.500 \times 10^{-3} \times 15 = \delta t$$

$$\boxed{7.501 \times 10^{-3} = \delta t}$$

A bar of 20mm diameter is tested in tension, It is observed that when a load of 40KN is applied, the Extension measured over a gauge length of 200mm is 0.12mm and Contraction in diameter is 0.0036mm. Find poisson's Ratio and Elastic Constants E, G and K.

Given:

Diameter = d = 20mm

Load = P = 40KN = 40×10^3 N

Length = L = 200mm

Change in Length = δl = 0.12mm

Change in diameter = δd = 0.0036mm.

To Find: Poisson's Ratio, $1/m$

Yang's Modulus, E

Modulus of Rigidity, G

Bulk modulus, K.

Sol:-

$$\text{poisson's Ratio } (\mu \text{ or } 1/m) = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}$$

$$\text{longitudinal Strain} = \frac{\delta l}{l} = \frac{0.12}{200} = 6 \times 10^{-4}$$

$$\text{lateral Strain} = \boxed{\frac{\delta d}{d} = \frac{0.0036}{20}} = 1.8 \times 10^{-4}$$

Poissons Ratio $(1/m) = \dfrac{1.8\times10^{-4}}{6\times10^{-4}} = 0.3$

$$\boxed{1/m = 0.3}$$

Young's Modulus, $E = \dfrac{\text{Stress } (\sigma)}{\text{Strain } (e)}$

$\because A = \frac{\pi}{4} d^2$

Stress, $\sigma = \dfrac{\text{load } (P)}{\text{Area } (A)}$

$= \dfrac{40\times10^3}{\frac{\pi}{4}\times d^2} = \dfrac{40\times10^3}{\frac{\pi}{4}\times20^2} = \dfrac{40\times10^3}{314.15} = 127.3 N/mm^2$

$E = \dfrac{\sigma}{e}$ ← longitudinal Strain $= \dfrac{127.3}{6\times10^{-4}} = 212.2\times10^3 N/mm^2$

$= 2.12\times10^5 N/mm^2$.

$\because E = 2G(1 + 1/m)$

$2.12\times10^5 = 2G(1+0.3)$

$\dfrac{2.12\times10^5}{1.3} = 2G$

$$\boxed{G = 81{,}538.4 \ N/mm^2}$$

$\because E = 3K(1 - 2/m)$

$2.12\times10^5 = 3K(1-(1/m)2)$

$2.12\times10^5 = 3K(1-(0.3)2)$

$$\boxed{K = 1.76\times10^5 \ N/mm^2}$$

A Square Steel rod 20mm x 20mm in Section, is to carry an axial load (compressive) of 100kN. Calculate the shortening in a length of 50mm. $E = 2.14 \times 10^8$ kN/m²

Given Data:

1m = 1000mm

Area, $A = 20mm \times 20mm = 0.02 \times 0.02 = 0.4 \times 10^{-4} m^2$
$= 0.0004 m^2$

length, $l = 50mm = 0.05m$

Load, $P = 100$ kN

$E = 2.14 \times 10^8$ kN/m²

Sol:

$\sigma = P/A$; $e = \frac{\delta l}{l}$; $E = \frac{\sigma}{e}$

Stress, $\sigma = \frac{P}{A} = \frac{100}{0.0004} = 250,000$ kN/m²

$E = \frac{Stress}{Strain} = \frac{\sigma}{e}$

$Strain = \frac{\sigma}{E} = \frac{250,000}{2.14 \times 10^8}$

$Strain = 1.1682 \times 10^{-3}$

$\frac{\delta l}{l} = 1.1682 \times 10^{-3}$

$\delta l = 1.1682 \times 10^{-3} \times 0.05$

$$\boxed{\begin{aligned}\delta l &= 5.841 \times 10^{-5} m \\ &= 0.00005841 m \\ &= 0.0584 mm.\end{aligned}}$$

A hollow cast-iron Cylinder 4m long, 300mm outer diameter, and thickness of metal 50mm is subjected to a central load on the top when standing straight. The stress produced is 75000 KN/m^2. Assume young's modulus for cast-iron as 1.5×10^8 KN/m^2 and find

i) Magnitude of the load
ii) longitudinal strain produced
iii) Total decrease in length

1m = 1000 mm
1mm = $\frac{1}{1000}$ m

Given data:

Outer diameter, $D = 300mm$
$= 0.3m$

Thickness, $t = 50mm = 0.05m$

Length, $l = 4m$

Stress produced, $\sigma = 75000\ KN/m^2$

$E = 1.5 \times 10^8\ KN/m^2$

d = inner diameter
t = thickness.

Outerdiameter, $D = d + 2t$

inner diameter, $d = D - 2t$
$= 0.3 - (2 \times 0.05)$
$= 0.2m$

Sol,

Hollow cylinder,
$Area = \frac{\pi}{4}(D^2 - d^2)$

* Magnitude of the load P:

$\sigma = P/A$

$P = \sigma \times A$
$= 75000 \times \frac{\pi}{4}(D^2 - d^2)$
$= 75000 \times \frac{\pi}{4}(0.3^2 - 0.2^2)$

$\boxed{P = 2945.2\ KN.}$

* Longitudinal Strain produced, e:

$E = \frac{Stress}{Strain} = \frac{\sigma}{e}$

$e = \frac{\sigma}{E} = \frac{75000}{1.5 \times 10^8} = 5 \times 10^{-4}$ (No unit)
$= 0.0005$ (No unit.

∠ Total decrease in length, δl

$$\text{Strain}(e) = \frac{\text{change in length}}{\text{original length}} = \frac{\delta l}{l}$$

$$0.0005 = \frac{\delta l}{4}$$

1m = 1000mm

$$\delta l = 0.0005 \times 4$$

$$= 2 \times 10^{-3} m$$

$$\boxed{\delta l = 2mm}$$

The following observations were made during a tensile test on a mild steel specimen 40mm in diameter and 200mm long.

Elongation with 40KN load (within limit of proportionality)

$\delta l = 0.0304$ mm

Yield load = 161KN

Maximum load = 242 KN

Length of specimen @ Fracture = 249 mm

Determine:

i) young's modulus of Elasticity

ii) yield point stress

iii) Ultimate Stress

iv) percentage Elongation

Sol's

i) youngs modulus of Elasticity E:

$$\text{Stress}, \sigma = P/A = \frac{40}{\pi/4 \times (0.04)^2} = 31.830 \times 10^3 \text{ KN/m}^2$$

$$= 3.18 \times 10^4 \text{ KN/m}^2$$

$$\text{Strain}, e = \frac{\delta l}{l} = \frac{0.0304}{200} = 1.52 \times 10^{-4} \text{ (no unit)}$$

$$= 0.00015 \text{ (No-unit)}.$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{3.18 \times 10^4}{0.00015} = 212 \times 10^6 \text{ KN/m}^2$$

$$= 2.12 \times 10^8 \text{ KN/m}^2.$$

ii) Yield point stress.

Yield point stress = $\frac{\text{Yield point load}}{\text{Area}}$

$$= \frac{161}{\frac{\pi}{4} \times (0.04)^2} = 128.11 \times 10^3 \ KN/m^2$$

$$= 12.8 \times 10^4 \ KN/m^2.$$

iii) Ultimate stress

Ultimate stress = $\frac{\text{Maximum load}}{\text{Area}}$

$$= \frac{242}{\pi/4 \times (0.04)^2} = 192.57 \times 10^3 \ KN/m^2$$

$$= 19.25 \times 10^3 \ KN/m^2$$

iv) percentage elongation

Percentage Elongation = $\frac{\text{Lengths of specimen @ fracture} - \text{original length}}{\text{Original length}}$

$$= \frac{249 - 200}{200} = 0.245 = 24.5\%.$$

A concrete cylinder of diameter 150mm and length 300mm when subjected to an axial compressive load of 240kN resulted in an increase of diameter by 0.127mm and a decrease in length of 0.28mm. Compute the value of poisson's ratio μ and modulus of Elasticity E.

Given data;

1m = 1000mm

Diameter of the cylinder, $d = 150mm = 0.15m$.

length of the cylinder, $l = 300mm = 0.3m$

Increase in diameter, $\delta d = 0.127mm (+)$

Decrease in length $\delta l = 0.28mm (-) = 0.00028mm$

Axial compressive load, $P = 240kN$

Sol;

$$\left.\begin{array}{l}\text{linear strain} \\ \text{or} \\ \text{longitudinal strain}\end{array}\right\} = \frac{\delta l}{l} = \frac{0.28}{300} = 9.33 \times 10^{-4}$$
$$= 0.000933$$

$$\left.\begin{array}{l}\text{transvers strain} \\ \text{or} \\ \text{laterial strain}\end{array}\right\} = \frac{\delta d}{d} = \frac{0.127}{150} = 8.466 \times 10^{-4}$$
$$= 0.000846$$

$$\text{poisson's Ratio } (\mu \text{ or } 1/m) = \frac{\text{laterial strain}}{\text{longitudinal strain}} = \frac{0.000846}{0.000933}$$

$$\boxed{\mu = 0.907}$$

$$\text{Stress, } \sigma = \frac{\text{load}}{\text{Area}} = \frac{P}{A} = \frac{240}{\pi/4 \times 0.15^2} = 13.58 \times 10^3 \, kN/m^2$$

$$E = \frac{Stress}{Strain} = \frac{13.58 \times 10^3}{0.000933} = 14.55 \times 10^6 \ KN/m^2$$

$$\boxed{E = 14.55 \ GN/m^2}$$

Q:- A steel bar 300mm long, 40mm wide and 25mm thick is subjected to a pull of 180 KN. Determine the change in volume of the bar. Take $E = 2 \times 10^5 \ N/mm^2$ & $1/m = 0.3$

Given data:

Length, $L = 300mm$; Width, $b = 40mm$

Thickness, $t = 25mm$

Pull, $P = 180 \ kN = 180 \times 10^3 N$

Young's modulus, $E = 2 \times 10^5 \ N/mm^2$

Poisson's Ratio, $1/m = 0.3$

$E = \frac{\sigma}{e}$

$e = \frac{\sigma}{E}$

$\frac{\delta L}{L} = \frac{\sigma}{E}$

Sol.

$$Volume = L \times b \times t$$
$$= 300 \times 40 \times 25$$
$$= 300 \times 10^3 \ mm^3 = 3 \times 10^5 \ mm^3$$

$$\sigma = \frac{P}{A} = \frac{180 \times 10^3}{b \times t} = \frac{180 \times 10^3}{40 \times 25} = 180 \ N/mm^2$$

$$\frac{\delta l}{L} = \frac{\sigma}{E} = \frac{180}{2 \times 10^5} = 9 \times 10^{-4} = 0.0009$$

$$e_v = \frac{\delta l}{l}(1 - 2\mu) = 0.0009 [1 - 2(0.3)]$$
$$= 3.6 \times 10^{-4}$$
$$= 0.00036.$$

$$e_v = \frac{dv}{V} = \frac{\delta l}{l}(1 - 2\mu)$$

$$\frac{dv}{V} = 0.00036$$

$$dv = 0.00036 \times V = 0.00036 \times 3 \times 10^5 = \boxed{108 \ mm^3 = dv}$$

A Steel Rod of 20mm diameter, 4m long carries a tensile load of 50kN. Calculate the change in length, diameter and Volume of the rod, Take $E = 2\times10^5\ N/mm^2$ and poisson's Ratio $= 0.3$.

Given data:

Diameter, $d = 20mm$

Length, $L = 4m = 4000mm$

Load, $P = 50kN = 50{,}000\ N$

Young's Modulus, $E = 2\times10^5\ N/mm^2$

poisson's Ratio, $1/m = 0.3$.

To Find,

* change in Length, δl
* change in diameter, δd
* Change in Volume, δV,

Sol.

$$\text{Stress} = \frac{P}{A} = \frac{50{,}000}{\pi/4 \times d^2} = \frac{50{,}000}{\pi/4 \times (20)^2} = 159.15\ N/mm^2$$

$$E = \frac{\sigma}{e} \; ; \; E = \frac{\sigma}{\frac{\delta l}{l}} \; \therefore \frac{\delta l}{l} = \frac{\sigma}{E} \; ; \; \delta l = \frac{\sigma}{E} \times l$$

$$\delta l = \frac{\sigma}{E} \times l$$

$$= \frac{159.15}{2\times10^5} \times 4000$$

$$\boxed{\delta l = 3.183\ mm}$$

$$\text{poisson's Ratio}, (1/m \text{ or } \mu) = \frac{\text{lateral Strain}}{\text{longitudinal Strain}} = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}$$

$$0.3 = \frac{\frac{\delta d}{20}}{\frac{3.183}{4000}}$$

$$0.3 = \frac{\delta d}{20} \times \frac{4000}{3.183}$$

$$\delta d = 4.77 \times 10^{-3}\,mm$$

$$\boxed{\delta d = 0.0047\,mm}$$

$$e_V = \frac{dv}{V} = \frac{\delta l}{l}(1-2\mu)$$

$$= \frac{3.183}{4000}[1-2(0.3)]$$

$$e_V = \frac{dv}{V} = 3.183 \times 10^{-4}$$

$$\text{Volume, } V = \frac{\pi}{4} d^2 \times l = \frac{\pi}{4}(20)^2 \times 4000$$

$$V = 1.25 \times 10^6\,mm^3$$

$$\frac{dv}{V} = 3.183 \times 10^{-4}$$

$$dv = 3.183 \times 10^{-4} \times 1.25 \times 10^6$$

$$\boxed{dv = 400\,mm^3}$$

Calculate the modulus of Rigidity and bulk modulus of a cylindrical bar of diameter 25mm and length 1.6m. If the longitudinal strain in a bar during tensile stress is four times the lateral strain, Determine the change in Volume, when the bar is subjected to a hydrostatic pressure of 100 N/mm². Take $E = 1 \times 10^5$ N/mm².

Given Data:-

Diameter, $d = 25$mm.

Length, $L = 1.6\text{m} = 1600\text{mm}$

Longitudinal Strain = 4 × Lateral Strain

$$e_l = 4 e_t$$

Stress, $\sigma = 100$ N/mm².

Young's modulus, $E = 1 \times 10^5$ N/mm²

To find:

Modulus of Rigidity, G.

Bulk Modulus, K

Change in Volume, dV.

Sol:

Poisson's Ratio, $1/m$ or μ = $\frac{\text{lateral strain}}{\text{Longitudinal Strain}}$

$$= \frac{e_t}{e_l} = \frac{e_t}{4e_t}$$

$$1/m = 1/4 = 0.25$$

$$\boxed{\text{Poisson's Ratio}, 1/m = 0.25}$$

Young's Modulus, $E = 2G(1 + 1/m)$

$$1 \times 10^5 = 2G(1 + 0.25)$$

$$1 \times 10^5 = 2.5\,G$$

$$\boxed{G = 4 \times 10^4 \ N/mm^2}$$

$E = 3K(1 - 2/m)$

$$1 \times 10^5 = 3K(1 - (1/m)2)$$

$$1 \times 10^5 = 3K(1 - (0.25)2)$$

$$1 \times 10^5 = 1.5(K)$$

$$\boxed{K = 6.6 \times 10^4 \ N/mm^2}$$

Bulk Modulus, $K = \dfrac{\text{Direct stress}}{\text{Volumetric strain}}$

$$K = \frac{\sigma}{e_v} \qquad \left\{ e_v = \frac{dv}{V} \right\}$$

$$6.6 \times 10^4 = \frac{\sigma}{\frac{dv}{V}}$$

$$6.6 \times 10^4 = \frac{100}{\frac{dv}{V}} \qquad \frac{dv}{V} = \frac{100}{6.6 \times 10^4}$$

$$\boxed{\frac{dv}{V} = 1.5 \times 10^{-3}}$$

Volume, $V = \dfrac{\pi}{4} d^2 L = \dfrac{\pi}{4}(25)^2 \times 1600$

$$\boxed{V = 7.85 \times 10^5 \ mm^3}$$

$$\frac{dv}{V} = 1.5 \times 10^{-3}$$

$$\frac{dv}{7.85 \times 10^5} = 1.5 \times 10^{-3} \qquad dv = 1.5 \times 10^{-3} \times 7.85 \times 10^5$$

$$\boxed{dv = 1177.5 \ mm^3}$$

A bar of cross section 8mm × 8mm is subjected to an axial pull of 8000N. The lateral dimension of the bar is found to be 7.996 × 7.996. If the modulus of Rigidity of the material is 9.6×10^4 N/mm². Determine the poisson's ratio and Modulus of Elasticity

Given Data:-

Area, $A = 8mm \times 8mm = 64 mm^2$

Axial pull, $P = 8000 N$

Laterial Dimension $= 7.996 \times 7.996 mm$

Modulus of Rigidity, $G = 9.6 \times 10^4 N/mm^2$

To Find:

Poisson's Ratio, $1/m$

Modulus of Elasticity, E

Sol:

Poisson's Ratio, $1/m = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

$$1/m = \frac{e_t}{e_l}$$

lateral strain, $e_t = \frac{\text{Change in lateral dimension}}{\text{Original dimension}}$

$$= \frac{8 - 7.996}{8}$$

$$\boxed{e_t = 0.0005}$$

Stress, $\sigma = \frac{P}{A} = \frac{8000}{64} = 125\ N/mm^2$

Young's Modulus $E = \frac{Stress}{Strain} = \frac{\sigma}{e_l}$

$$e_l = \frac{\sigma}{E} = \frac{125}{E}$$

$$\boxed{e_l = \frac{125}{E}}$$

$$1/m = \frac{e_t}{e_l} = \frac{0.0005}{\frac{125}{E}}$$

$$\frac{1}{m} = \frac{0.0005}{1} \times \frac{E}{125}$$

$$\frac{1}{m} = 4 \times 10^{-6} E$$

$$\frac{1}{4 \times 10^{-6}} = Em$$

$$\boxed{Em = 2,50,000}$$

Modulus of Rigidity, $G = 9.6 \times 10^4\ N/mm^2$

$$E = 2G(1 + 1/m)$$

$$E = 2G\left(\frac{m+1}{m}\right)$$

$$Em = 2G(m+1)$$

$$2,50,000 = 2G(m+1)$$

$$2,50,000 = 2 \times 9.6 \times 10^4 (m+1)$$

$$m+1 = 1.30$$

$$m = 0.30$$

$$\boxed{\text{Poisson's Ratio},\ 1/m = 3.3}$$

$$mE = 2,50,000$$

$$0.30(E) = 2,50,000$$

$$\boxed{E = 8.3 \times 10^5\ N/mm^2}$$

For a given material, Young's Modulus is $1 \times 10^5 \, N/mm^2$ and Modulus of rigidity is $4 \times 10^4 \, N/mm^2$. Find the bulk modulus and lateral contraction of a round bar of 50mm diameter and 2.5m long, when length is increased 2.5m

Given data:

Young's Modulus, $E = 1 \times 10^5 \, N/mm^2$

Modulus of Rigidity, $G = 4 \times 10^4 \, N/mm^2$

Diameter, $d = 50mm$

Length, $l = 2.5m = 2500mm$

Change in length, $\delta l = 2.5mm$

To find:

* Bulk Modulus, K
* Lateral Contraction of Round bar, δd

Sol:

$$E = 3K(1 - 2/m)$$

$$E = 3K(1 - (1/m)2)$$

$$1 \times 10^5 = 3K(1 - (1/m)2) \longrightarrow (1)$$

Poisson's Ratio, $1/m = \frac{\text{lateral Strain}}{\text{longitudinal Strain}}, \frac{e_t}{e_l}$

$$1/m = \frac{e_t}{(e_l)}$$

longitudinal Strain, $e_l = \frac{\delta l}{l} = \frac{2.5}{2500} = \boxed{1 \times 10^{-3} = e_l}$

$$E = 2G\left(1 + \frac{1}{m}\right)$$

$$1\times10^{5} = 2\times4\times10^{4}\left(1+\frac{1}{m}\right)$$

$$1+\frac{1}{m} = 1.25$$

$$\boxed{\frac{1}{m} = 0.25}$$

$$\frac{1}{m} = \frac{e_t}{e_l}$$

$$0.25 = \frac{e_t}{1\times10^{-3}}$$

$$\boxed{e_t = 2.5\times10^{-4}}$$

lateral strain, $e_t = \frac{\delta d}{d}$

$$2.5\times10^{-4} = \frac{\delta d}{50}$$

$$\boxed{\delta d = 0.0125\,mm}$$

From Eq(1) $1\times10^{5} = 3K\left(1-\left(\frac{1}{m}\right)2\right)$

$$1\times10^{5} = 3K\left(1-(0.25)2\right)$$

$$\boxed{K = 6.6\times10^{4}\ N/mm^2}$$

Uni-2

If the modulus of Elasticity of a material is $200 \, GN/m^2$ and modulus of Rigidity is $80 \, GN/m^2$. Determine the Poisson's Ratio and bulk modulus.

Given data:

Modulus of Elasticity, $E = 200 \, GN/m^2$

$= 200 \times 10^9 \, N/m^2$

Modulus of Rigidity, $G = 80 \, GN/m^2$

$= 80 \times 10^9 \, N/m^2$

To Find;

Poisson's Ratio ($1/m$ or μ)

Bulk Modulus (K).

Sol.

$E = 2G(1 + 1/m)$

$200 \times 10^9 = 2(80 \times 10^9)[1 + 1/m]$

$\frac{200 \times \cancel{10^9}}{160 \times \cancel{10^9}} = 1 + 1/m$

$1.25 = 1 + 1/m$

$1/m = 1.25 - 1$

$\boxed{1/m = 0.25}$

$E = 3K(1 - 2/m)$

$E = 3K[1 - (1/m)2]$

$200 \times 10^9 = 3[K][1 - (0.25)2]$

$\frac{200 \times 10^9}{1.5} = K$

$K = 1.33 \times 10^{11} \, N/m^2$

$\boxed{K = 133 \times 10^9 \, N/m^2}$

A steel rod 6m long and 35mm in diameter is subjected to an axial tensile load of 60kN. Determine the change in diameter, length and volume of the rod. Assume Modulus of Elasticity as 2×10^5 N/mm^2 and poisson's ratio = 0.26.

Given data:

Diameter = 35mm

Length, L = 6m = 6000mm

Load, P = 60kN = 60×10^3 N

Young's modulus $E = 2\times10^5$ N/mm^2

poisson's Ratio, $1/m = 0.3$

To Find:

* Change in Length, δl
* Change in diameter, δd
* Change in Volume, δV.

Sol.

$$\text{Stress} = \frac{P}{A} = \frac{60\times10^3}{\frac{\pi}{4}(35)^2} = \frac{60\times10^3}{962.11} \doteq 62.36 \text{ N/mm}^2$$

$$E = \frac{\sigma}{e}; \quad E = \frac{\sigma}{\frac{\delta l}{l}}; \quad \frac{\delta l}{l} = \frac{\sigma}{E}; \quad \delta l = \frac{\sigma}{E}\times l$$

$$\delta l = \frac{\sigma}{E}\times l$$

$$= \frac{62.36}{2\times10^5}\times 6000 = \boxed{1.87\text{mm} = \delta l}$$

$$\text{poisson's Ratio, } (1/m \text{ or } \mu) = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}.$$

$$0.3 = \frac{\delta d}{d}\times\frac{l}{\delta l}; \quad 0.3 = \frac{\delta d}{35}\times\frac{6000}{1.87}$$

$$\boxed{\delta d = 0.0032\text{mm}}$$

$$e_v = \frac{dv}{V} = \frac{\delta l}{l}\left[1 - 2\mu\right]$$

$$\frac{dv}{V} = \frac{1.87}{6000}\left[1 - 2(0.3)\right] = 1.246 \times 10^{-4}$$

Volume, $V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} \times (35)^2 \times 6000$

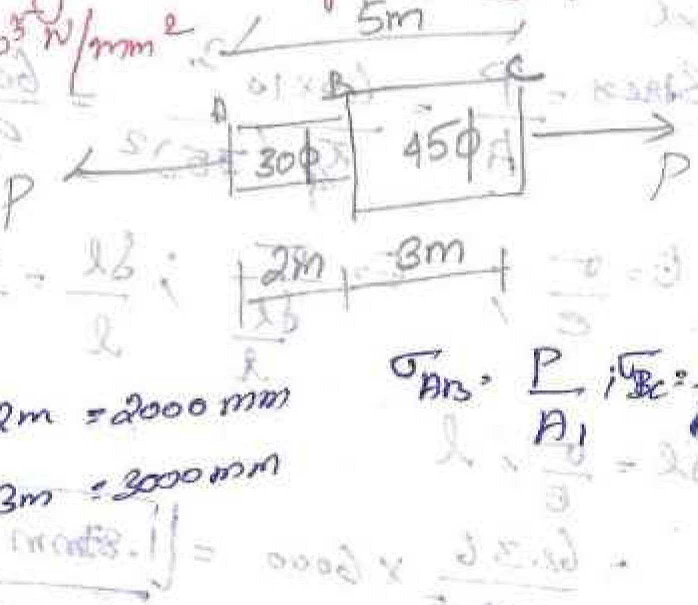
$$= 5.77 \times 10^6 \, mm^3$$

$$\frac{dv}{V} = 1.246 \times 10^{-4}$$

$$dv = 1.246 \times 10^{-4} \times 5.77 \times 10^6$$

$$\boxed{dv = 719.326 \, mm^3}$$

A circular rod is subjected to a axial tensile load of 40kN is 5m length. The cross section of the rod has a diameter 30mm for a length of 2m and has diameter 45mm for the rest of the length. Find the total change in length of the rod assuming Modulus of Elasticity of the material of rod as $1.5 \times 10^5 \, N/mm^2$.



Given data:

load $= 40 \, kN = 40 \times 10^3 \, N$

length $= 5m = 5000 \, mm$

$E = 1.5 \times 10^5 \, N/mm^2$

when $\phi = 30mm$; length $= 2m = 2000 \, mm$

$\phi = 45mm$; length $= 3m = 3000 \, mm$

$\sigma_{AB} = \frac{P}{A_1}$; $\sigma_{BC} = \frac{P}{A_2}$

To find: δl

Sol: $\sigma = P/A$; $e = \frac{\delta l}{l}$, $E = \frac{\sigma}{e}$

$E = \frac{P/A}{e}$; $\rightarrow E = \frac{P}{Ae}$; $e = \frac{P}{AE}$; $\frac{\delta l}{l} = \frac{P}{AE}$

$$\delta l = \frac{Pl}{AE}$$

$\delta l = \frac{Pl}{AE}$, $\delta l_1 = \frac{Pl_1}{A_1 E}$, $\delta l_2 = \frac{Pl_2}{A_2 E}$

Total Elongation = $\delta l_1 + \delta l_2$

$$\delta l = \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E}$$

$$= \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right]$$

$$= \frac{40\times10^3}{1.5\times10^5}\left[\frac{2000}{\frac{\pi}{4}(30)^2} + \frac{3000}{\frac{\pi}{4}(45)^2}\right]$$

$$= 0.26\left[2.829 + 1.886\right]$$

$$= 0.26(4.715)$$

$$\boxed{\delta l = 1.225\,mm}.$$

The Value of Modulus of Rigidity for a Specimen is $0.5\times10^5\,N/mm^2$. A 20mm diameter rod of this material was Subjected to tensile load of 30KN and the change in diameter was 4×10^{-3}mm. Calculate poisson's ratio and Modulus of Elasticity.

Given data:-

$G = 0.5\times10^5\,N/mm^2$

$\phi = 20mm$. Area $(a) = \frac{\pi}{4}\times 20^2$

$= 314.15\,mm^2$

$P = 30KN = 30\times10^3\,N$

$\delta d = 4\times10^{-3}\,mm$.

To find: $\mu = ?$

$E = ?$

Stress $(\sigma) = \frac{P}{A} = \frac{3\times10^3}{314.15} = 95.49\ N/mm^2$

lateral strain, $(e_t) = \frac{\delta d}{d} = \frac{4\times10^{-3}}{20} = 2\times10^{-4}$
$= 0.0002$

$E = \frac{\sigma}{e}$; $e = \frac{\sigma}{E} = \boxed{\frac{95.49}{E} = e}$

$\mu = \frac{\text{lateral strain}}{\text{long. strain}}$

lateral strain $= \mu \times$ long. strain

$\boxed{0.0002 = \mu \times \frac{95.49}{E}}$

$E = \frac{\mu \times 95.49}{0.0002} \rightarrow (1)$

$E = 2G[1+\mu]$

$G = \frac{E}{2[1+\mu]}$

$0.5\times10^5 = \frac{E}{2[1+\mu]}$

$1\times10^5 = \frac{E}{[1+\mu]}$

$\boxed{E = 1\times10^5[1+\mu]} \longrightarrow (2)$

By solving Eq (1) & (2)

$1\times10^5[1+\mu] = \mu \times \frac{95.49}{0.0002}$

$1\times10^5[1+\mu] = 477.45\times10^3(\mu)$

$\frac{1+\mu}{\mu} = \frac{477.45\times10^3}{1\times10^5}$

$\frac{1+\mu}{\mu} = 4.77$

$\frac{1}{\mu} + \frac{\mu}{\mu} = 4.77$

$\frac{1}{\mu} + 1 = 4.77$

$\frac{1}{\mu} = 3.77$

$\boxed{\mu = 0.265}$

Sol μ in (1)

$E = \frac{\mu \times 95.49}{0.0002}$

$= \frac{95.49\times0.265}{0.0002}$

$\boxed{E = 1.26\times10^5\ N/mm^2}$

A cube with each side 300mm is subjected to a tensile force of 200kN normal to all of the faces. Calculate the change in Volume if poisson's Ratio is 0.25 and modulus of Elasticity of material of cube is 2×10^5 N/mm². What should be the change done to the force acting normal to upward side so that no change in Volume occurs?

Area = 90000mm²

Volume = 27×10^6 mm³

Given data:

Cube size = 300 × 300 × 300 mm

Load = 200 kN = 200×10^3 N

poisson's Ratio ($1/m$ or μ) = 0.25

Modulus of Elasticity, E = 2×10^5 N/mm².

Sol:-

$$\text{Stress } (\sigma) = \frac{P}{A} = \frac{200\times10^3}{300\times300} = 2.22 \text{ N/mm}^2$$

$$E = \frac{\sigma}{e} \; ; \; e = \frac{\sigma}{E} = \frac{2.22}{2\times10^5} = \boxed{1.11\times10^{-5} = e} = \frac{\delta l}{l}$$

$$e_v = \frac{dv}{V} = \frac{\delta l}{l}[1-2\mu]$$

$$dv/v = 1.11\times10^{-5}\,[1-2(0.25)] = 5.55\times10^{-6}$$

$$dv = 1.11\times10^{-5}\,[0.5]\times27\times10^6$$

$$dv = 149.85 \simeq 150 \text{ mm}^3$$

$$\boxed{dv = 150 \text{ mm}^3}$$

$$E = 3K[1-2/m) \; ; \; 2\times10^5 = 3K[1-(0.25)2]$$

$$\boxed{K = 133.33\times10^3 \text{ N/mm}^2}$$

Bulk modulus, $K = \frac{\text{Direct Stress}}{\text{Vol. Strain}} = \frac{\sigma}{e_v}$

$$133.33\times10^3 = \frac{\sigma}{5.55\times10^{-6}}$$

$$\sigma = 133.33\times10^3\times5.55\times10^{-6}$$

$$\boxed{\sigma = 0.739 \text{ N/mm}^2}$$

$$\sigma = \frac{P}{A} \; ; \; P = \sigma\times A = 0.739\times300\times300$$

$$P = 66.60\times10^3 \text{ N (or) } 66.60 \text{ kN}.$$

The safe stress for a hollow steel column which carries an axial load of 2.1×10^3 KN is $125\ MN/m^2$. If the external diameter of the column is 30cm, determine the internal diameter.

Given data:

Safe stress $\sigma = 125\ MN/m^2 = 125 \times 10^6\ N/m^2$

Axial load $P = 2.1 \times 10^3\ KN = 2.1 \times 10^6\ N$

External diameter, $D = 30cm = 0.30m$

To find:

Internal diameter.

Sol:

Area of cross-section of the column

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(0.30^2 - d^2)\ m^2$$

$$\sigma = \frac{P}{A} \qquad 125 \times 10^6 = \frac{2.1 \times 10^6}{\frac{\pi}{4}(30^2 - d^2)}$$

$$0.30^2 - d^2 = \frac{2.1 \times 10^6}{0.785 \times 125 \times 10^6}$$

$$0.30^2 - d^2 = 2.1 / 98.125$$

$$0.30^2 - 0.021 = d^2$$

$$d = 0.2626\ m$$

$$\boxed{d = 26.26\ cm}$$

A circular steel bar of 10 mm diameter and 100mm gauge length is tested under tension. A tensile force of 10KN increases its length by 0.06mm while the diameter is decreased by 0.0018mm. Determine (i) young's modulus of Elasticity (ii) poisson's Ratio for the material of the bar.

Tensile force $P = 10\,kN = 10{,}000\,N$

Area of cross-section, $A = \pi/4\,(10)^2 = 78.54\,mm^2$

$D = 10\,mm$

Original length, $L = 100\,mm$

$\delta d = 0.0018\,mm$

Change in length $= \delta l = 0.06\,mm$

$\sigma = P/A \qquad e = \delta l / l \qquad E = \frac{\sigma}{e} = \frac{P/A}{\delta l / l} \qquad E = \frac{P}{A} \times \frac{l}{\delta l}$

$$E = \frac{10{,}000 \times 100}{78.54 \times 0.06} = \boxed{212 \times 10^3\,N/mm^2 = E}$$

$$e_{long} = \frac{\delta l}{l} = \frac{0.06}{100} = 0.0006$$

$$e_{lat} = \frac{\delta d}{d} = \frac{0.0018}{10} = 1.8 \times 10^{-4}$$

$$(1/m \text{ or } \mu)\ \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{1.8 \times 10^{-4}}{0.0006} = \boxed{0.3 = 1/m}$$

A stepped circular bar having diameter 20mm, 15mm and 10mm is subjected to over axial length of 100mm, 80mm and 60mm is subjected to an axial tensile force of 5kN. If $E = 100 \times 10^3\,N/mm^2$ and $1/m = 0.3$ for the material of the bar, determine.

(a) Total change in length

(b) Change in each diameter.

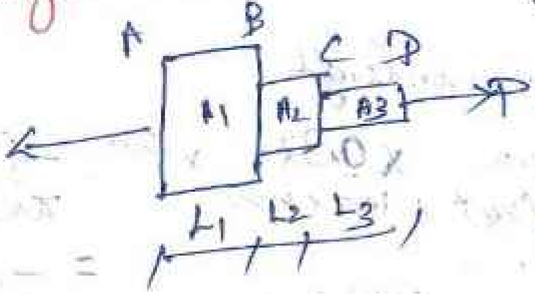
$E = \frac{\sigma}{e}$

$E = \frac{P/A}{e}$

$e = \frac{P}{AE}$

$\frac{\delta l}{l} = \frac{P}{AE}$

$\delta l = \frac{Pl}{AE}$



$\sigma_{AB} = \frac{P}{A_1} \qquad \delta l_1 = \frac{PL_1}{A_1 E}$

$\sigma_{BC} = \frac{P}{A_2} \qquad \delta l_2 = \frac{Pl_2}{A_2 E}$

$\sigma_{CD} = \frac{P}{A_3} \qquad \delta l_3 = \frac{Pl_3}{A_3 E}$

Total Elongation $= \delta l_1 + \delta l_2 + \delta l_3$

$$\delta l = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

$$\delta l = \frac{P}{E}\left[L_1/A_1 + L_2/A_2 + L_3/A_3\right]$$

given data:-

length : $l_1 = 100mm$, $l_2 = 80mm$; $l_3 = 60mm$

Diameter $D_1 = 20mm$, $D_2 = 15mm$ $D_3 = 10mm$

$P = 5kN = 5000N$

$E = 100\times10^3 \ N/mm^2$

$A_1 = \frac{\pi}{4}(20^2) = 0.785\times20^2 = 314.16 \ mm^2$

$A_2 = \frac{\pi}{4}\times(15)^2 = 176.62 \ mm^2$

$A_3 = \frac{\pi}{4}\times(10)^2 = 78.5 \ mm^2$

$$\delta l = \frac{P}{E}\left[l_1/A_1 + l_2/A_2 + l_3/A_3\right]$$

$$= \frac{5000}{100\times10^3}\left[\frac{100}{314.16} + \frac{80}{176.62} + \frac{60}{78.5}\right]$$

$$= 0.05\left[0.318 + 0.453 + 0.764\right]$$

$$\boxed{\delta l = 0.076mm}$$

$$\boxed{\frac{\delta l}{l} = \frac{P}{AE}}$$

$$\left(\frac{1}{m}\right) \text{ or } \mu = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}$$

$$\Rightarrow \frac{\delta l}{l}\times\mu = \frac{\delta d}{d}$$

$$\frac{\delta l}{l} = \frac{\delta d}{d}\times\frac{1}{\mu}$$

$$\frac{\delta d}{d}\times\frac{1}{\mu} = \frac{P}{AE}$$

$$\delta d = \frac{P}{AE}\times d\mu$$

decrease in length

$$\delta d_1 = -\frac{5000}{\frac{\pi}{4}(20)^2\times100\times1000}\times0.32\times\cancel{20} = \frac{-4\times5000\times0.32}{\pi\times20\times100\times1000}$$

$$= \frac{-6400}{6.28\times10^6}$$

$$\boxed{\delta d_1 = -1.018\times10^{-3}mm}$$

$$\delta d_2 = -\frac{4\times5000\times0.32}{\pi\times15\times100\times1000}$$

$$\boxed{\delta d_2 = -1.358\times10^{-3}mm}$$

$$\delta d_3 = -\frac{4\times5000\times0.32}{\pi\times10\times100\times1000}$$

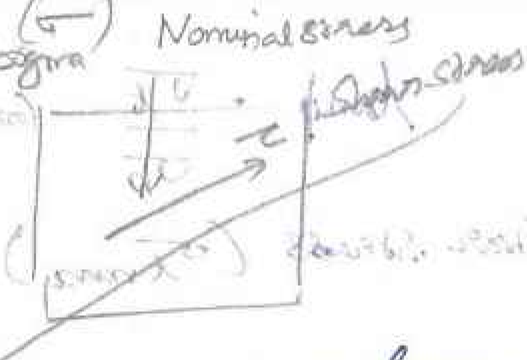
$$\boxed{\delta d_3 = -2.037\times10^{-3}mm}$$

σ_1

σ_2

σ_3

Principal Stresses:



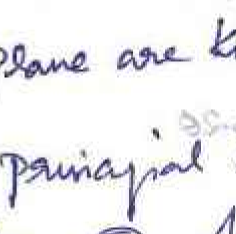
The plane, which have no shear stress are known as principal planes, hence principal plane are the plane of zero shear stress. These plane carry only Normal stress.

The Normal stress acting on a principal plane are known as principal stresses.

principal stresses and principal planes are determined by
- Analytical Method
- Graphical Method. (Mohr Circle).

I Normal stress (σ_n)

II Shear stress or Tangential. (τ)



$\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$\sigma_n = \sigma \cos^2\theta$

σ_t or $\tau = \frac{\sigma}{2} \sin 2\theta$

Stress are Subjected to principal direction:-

σ_1, σ_2, θ

Normal stress $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$

Shear (or) tangential Stress (τ) or $(\sigma_t) = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$

Resultant Stress $(\sigma_R) = \sqrt{\sigma_n^2 + \tau^2}$ ($\tau^2 \to (\sigma_t)^2$)

Obliquity (ϕ)

The angle made by the resultant stress with the normal of the oblique plane is known as Obliquity.

Tangential Stress Shear Stress

$$\tan\phi = \frac{\sigma_t}{\sigma_n} \quad (\text{or}) \quad \frac{\tau}{\sigma_n}$$

Max Shear Stress ($\sigma_{t\,max}$) or (τ_{max}) $= \dfrac{\sigma_1 - \sigma_2}{2}$

Stress subjected to principle direction accompanied by Shear stress: $\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau \\ \theta \end{bmatrix}$

$$\text{Normal stress } (\sigma_n) = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\text{Shear Stress } (\sigma_t) \text{ or } (\tau) = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta.$$

$$\text{Resultant Stress } (\sigma_R) = \sqrt{\sigma_n^2 + \sigma_t^2} \ (\text{or}) \ (\tau)^2$$

Principle Stress:

$$\sigma_1, \sigma_2 = \frac{\sigma_1 + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$\tau_{max} \text{ or } \sigma_{t\,max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

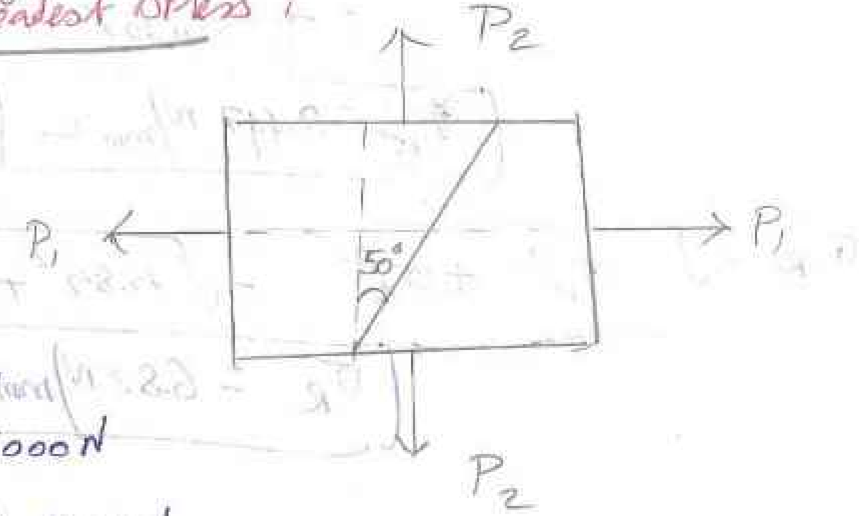
or

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}.$$

$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau \end{pmatrix}$

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

A 5mm thick aluminium plate has a width of 300mm and a length of 600mm subjected to pulls of 15000N & and 9000N respectively in axial and transverse direction. Determine the normal, tangential and resultant stresses on a plane 40° to the greatest stress.



Given data:

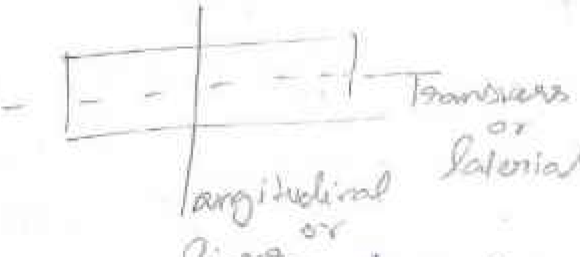
Width, $b = 300mm$

Length, $l = 600mm$

Thickness, $t = 5mm$

Axial load, $P_1 = 15000N$

Transverse load, $P_2 = 9000N$



To find:-

Normal stress (σ_n)

Tangential stress (σ_t)

Resultant stress (σ_R) on a plane 40° to the greatest stress

$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \theta \end{bmatrix}$

Sol

Axial stress, $\sigma_1 = \dfrac{\text{Axial load}}{\text{Area}} = \dfrac{15000}{5 \times 300} = 10 \ N/mm^2$

Transverse stress, $\sigma_2 = \dfrac{\text{Transverse load}}{\text{Area}} = \dfrac{9000}{600 \times 5} = 3 N/mm^2$

Maximum stress is σ which is horizontal

$\theta = 90° - 40° = 50°$ to the vertical

Normal stress, $\sigma_n = \dfrac{\sigma_1 + \sigma_2}{2} + \dfrac{\sigma_1 - \sigma_2}{2} \cos 2\theta$

$= \dfrac{10+3}{2} + \dfrac{10-3}{2} \cos 2 \times 50°$

$= 13/2 + 7/2 \cos 100° = 6.5 + (-0.608)$

$= 6.5 - 0.608$

$\boxed{\sigma_n = 5.89 \ N/mm^2}$

Trangential Stress, $\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \times \sin(2\theta)$

$= \frac{10-3}{2} \sin(2 \times 50°)$

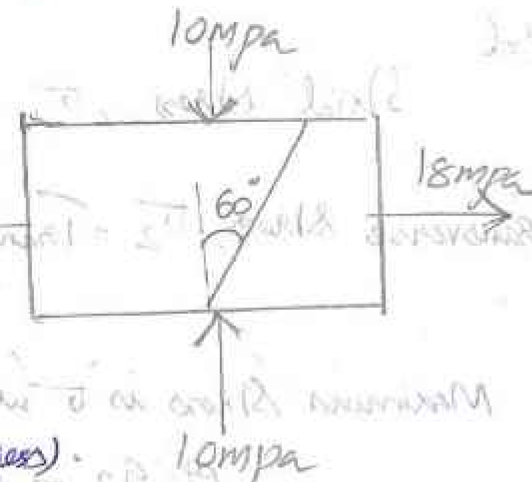
$= 7/2 \sin 100°$

$\boxed{\sigma_t = 3.447 \ N/mm^2}$

$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{5.89^2 + 3.447^2}$

$\boxed{\sigma_R = 6.82 \ N/mm^2}$

The principal stresses @ a point in the section of a heat exchanger shell are 18Mpa (tensile) and 10mpa (compressive) acting mutually perpendicular to each other. Determine the normal, shear and resultant stress intensities on a plane whose normal is inclined @ 60° to 10mpa stress. Find the maximum shear stress.



Given:

Pa ⇒ N/m^2

Mpa ⇒ N/mm^2

$\sigma_1 = 18$ Mpa
$= 18 \ N/mm^2$

$\sigma_2 = -10$ Mpa
$= -10 \ N/mm^2$ (compressive stress)

$\theta = 60°$

To find: Normal (σ_n) stress,
Shear (σ_t) stress,
Resultant stress (σ_R)
Maximum shear stress ($\sigma_{t max}$) or (τ_{max})

Solution:

Normal Stress, $(\sigma_n) = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2} \cos 2\theta$

$= \frac{18-10}{2} + \frac{18-(-10)}{2} \cos(2\times 60^\circ)$

$= \frac{8}{2} + \frac{28}{2}(-0.5)$

$= 4 - 7$

$\boxed{\sigma_n = -3 N/mm^2}$

Tangential Stress $(\sigma_t) = \frac{\sigma_1-\sigma_2}{2} \times \sin 2\theta$

$= \frac{18-(-10)}{2} \sin(2\times 60^\circ)$

$= \frac{28}{2}(0.866)$

$\boxed{\sigma_t = 12.12 N/mm^2}$

Resultant Stress, $(\sigma_R) = \sqrt{\sigma_n^2 + (\sigma_t)^2}$

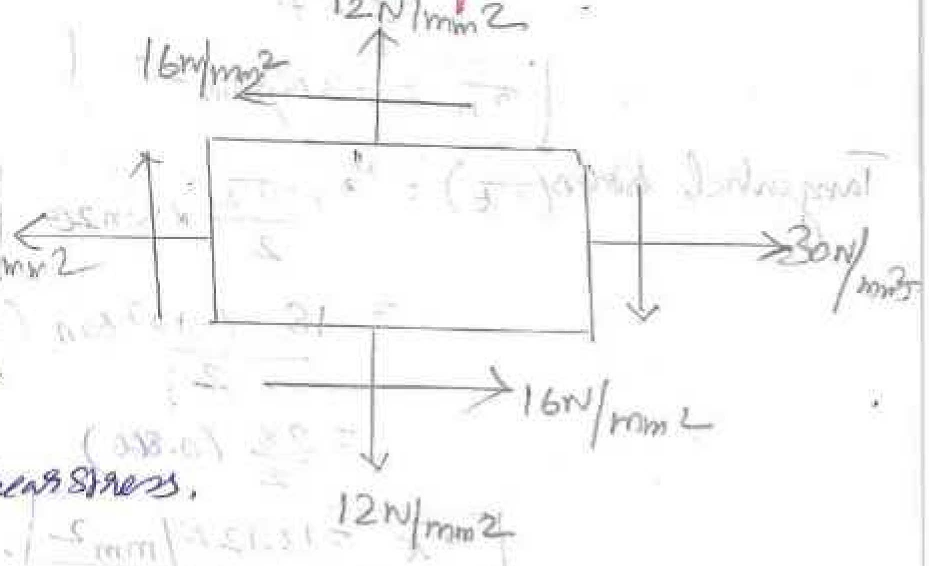
$= \sqrt{(-3)^2 + (12.12)^2} = \sqrt{9 + 146.89}$

$\boxed{\sigma_R = 12.485 N/mm^2}$

Maximum Shear Stress $(\sigma_{t\,max}) = \frac{\sigma_1-\sigma_2}{2}$

$(\sigma_{t\,max}) = \frac{18-(-10)}{2} = \frac{28}{2} = 14 N/mm^2$

At a point in a Strained body Subjected to two mutually Perpendicular normal tensile stresses of magnitude 30 mpa and 12 Mpa accompanied by a Shear stress of 16 Mpa. Locate the principal planes and Evaluate the principal stresses. Also Calculate the maximum intensity of shear stress and Specify its planes.



Given data

$\sigma_1 = 30 Mpa = 30 N/mm^2$

$\sigma_2 = 12 Mpa = 12 N/mm^2$

$q = 16 Mpa = \tau$ = Shear stress.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau \end{bmatrix}$$

Sol: $\tan 2\theta = \dfrac{2\tau}{\sigma_1 - \sigma_2}$

$$2\theta = \tan^{-1}\left(\frac{2 \times 16}{30 - 12}\right) = \tan^{-1}\left(\frac{32}{18}\right)$$

$$2\theta = \tan^{-1}(1.778)$$

$$2\theta = 60^\circ 64'$$

$$\theta = 30^\circ 32'$$

Major principal stress $= \dfrac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\dfrac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{30 + 12}{2} + \sqrt{\left(\frac{30 - 12}{2}\right)^2 + 16^2}$$

$$= 21 + \sqrt{81 + 256}$$

$$= 39.56 \ N/mm^2$$

Minor principal stress

$$= \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{30+12}{2} - \sqrt{\left(\frac{30-12}{2}\right)^2 + 16^2}$$

$$= 21 - \sqrt{81+256}$$

$$= 21 - 18.36$$

$$= 2.642 \ N/mm^2$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(30-12)^2 + 4(16)^2}$$

$$= \frac{1}{2}\sqrt{324+1024}$$

$$\boxed{\tau_{max} = 18.36 \ N/mm^2}$$

A point in a strained material is subjected to two mutually perpendicular tensile stresses of 200 N/mm2 and 1.00 N/mm2. Determine the intensities of normal, tangential and resultant stresses on a plane inclined at 30° anti-clockwise to the axis of the minor stress.

Given:

$\sigma_1 = 200 \ N/mm^2$ $\quad \sigma_2 = 100 \ N/mm^2$ $\quad \theta = 30°$ @ minor stress

To Find: Normal (σ_n) stress, Resultant stress (σ_R), Tangential stress (σ_t)

Sol:

Normal stress (σ_n) $= \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta$

$$= \frac{200+100}{2} + \frac{200-100}{2}\cos 2(30°)$$

$$= 150 + 50\cos(60°)$$

$$(\sigma_n) = 175 \ N/mm^2$$

Tangential Stress $(\sigma_t) = \frac{\sigma_1 - \sigma_2}{2} \times \sin 2\theta$

$= \frac{200 - 100}{2} \sin 2(30^\circ)$

$= 50 \sin 60^\circ$

$\boxed{\sigma_t = 43.30\ N/mm^2}$

Resultant Stress $(\sigma_R) = \sqrt{\sigma_n^2 + \sigma_t^2}$

$= \sqrt{175^2 + 43.30^2}$

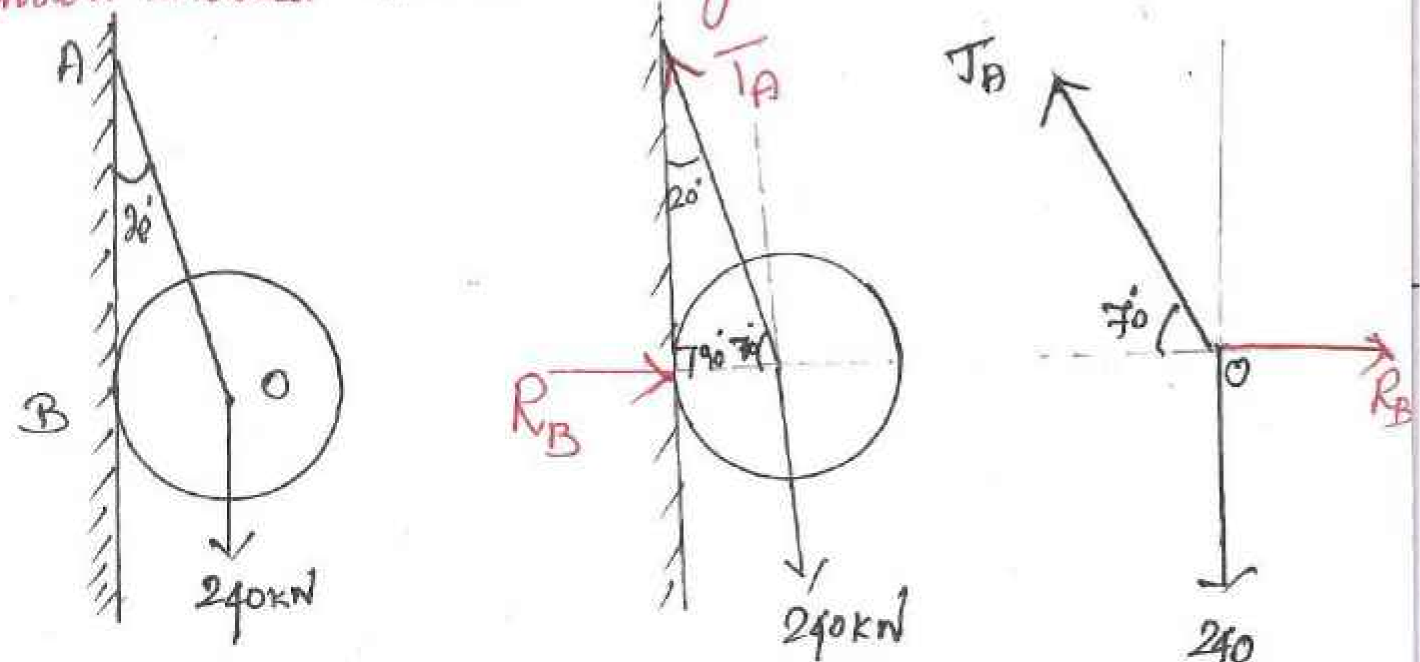
$\boxed{\sigma_R = 180.27\ N/mm^2}$

Maximum Shear Stress $(\sigma_{t\,max}) = \frac{\sigma_1 - \sigma_2}{2}$

$= \frac{200 - 100}{2}$

$\boxed{\sigma_{t\,max} = 50\ N/mm^2}$

A horizontal cylinder of weight 240kN is held against a wall with the help of string as shown in fig. The string makes an angle 20° with the wall. Determine the reaction of the wall and tension induced in the string.



By Lami's Theorem

$$\frac{T}{\sin 90^\circ} = \frac{R_B}{\sin(90+70^\circ)} = \frac{240}{\sin(20+90^\circ)}$$

$$\frac{T}{\sin 90^\circ} = \frac{240}{\sin 110^\circ}$$

$$T = \sin 90^\circ \times \frac{240}{\sin 110^\circ}$$

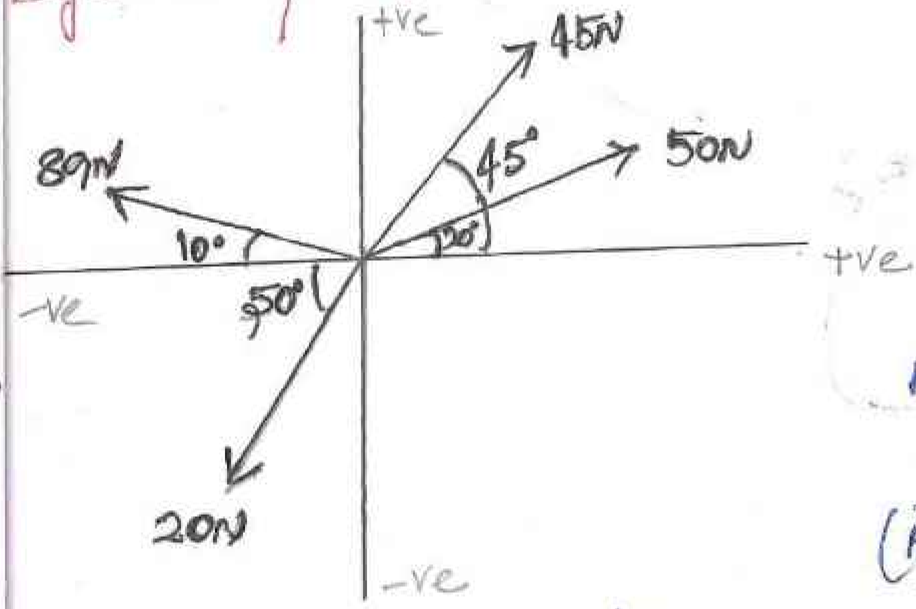
$$\boxed{T = 255.40\ kN}$$

$$\frac{R_B}{\sin 160^\circ} = \frac{240}{\sin 110^\circ}$$

$$R_B = \sin 160^\circ \times \frac{240}{\sin 110^\circ}$$

$$\boxed{R_B = 87.35\ kN}$$

Four forces 50N, 45N, 89N and 20N act@ angle 20°, 45°, 170° and 230° to a fixed line such that all the forces act towards a same point. What will be the magnitude and direction of a force which will bring the above system to equilibrium ?



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(-21.7)^2 + (49.04)^2}$

$\boxed{R = 53.63 N}$

Magnitude of the resultant force

Resolving all the forces horizontally.

$\Sigma H = 50\cos 20° + 45\cos 45° - 89\cos 10° - 20\cos 50°$

$= 46.98 + 31.81 - 87.64 - 12.85$

$\boxed{\Sigma H = -21.7N}$

And now resolving the all forces vertically

$\Sigma V = 50\sin 20 + 45\sin 45° + 89\sin 10° - 20\sin 50°$

$= 17.10 + 31.81 + 15.45 - 15.32$

$\boxed{\Sigma V = 49.04 N}$

$\tan\theta = \dfrac{\Sigma V}{\Sigma H} = \dfrac{49.04}{-21.7} = -2.25$ or $\theta = 66.03°$

Since ΣH is negative & ΣV is positive, therefore resultant lies between 180° to 90° or (90° and 180°)

$\theta = 180° - 66.03 = 113.97°$